Developing Higher-Order Thinking Skills of Mathematics Education Students: A Grounded Theory

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The purpose of this study was to produce a theoretical formula for developing the higher-order thinking skills of mathematics students. This is a qualitative study that applies grounded theory to produce and develop a theory based on the empirical data collected by the researcher. The subjects of this study were mathematics education students at the Faculty of Teacher Training and Education, Nusa Cendana University, Indonesia by considering gender demographics and mathematics skills. The written data was analysed qualitatively using the indicators of higher-order thinking skills. Consequently, an in-depth interview was conducted to explore the subject's writing ability and thinking habits, and the factors or issues relevant to the development of the subject's higher-order thinking skills. The process of theory formation was conducted through the stages of open coding, axial coding and selective coding. The stages of data analysis resulted in an empirical theory of developing higher-order thinking skills. This involved: 1) building motivation and confidence; 2) contextual learning (cultural context); 3) the habit of asking scientific question, 4) group discussion; 5) higher-order thinking exercises; 6) routine discussions in the family and community; and 7) learning to modify questions. The seven points describe the whole process and the support the personal self, lecturers and parents.

\textbf{Key words:} Grounded theory, Higher-order thinking.
Introduction

The concept of higher-order thinking was proposed by Bloom, Englehart, Furst, Hill, and Krathwohl (1956) in the taxonomy of education. They divided the cognitive level into 6 parts, namely (1) knowledge; (2) comprehension; (3) application; (4) analysis; (5) synthesis, and (6) evaluation. Anderson and Krathwohl (2001) revised Bloom's taxonomy into: (1) remembering; (2) understanding; (3) applying; (4) analysing; (5) evaluating, and (6) creating. Higher-order thinking levels in the new taxonomy are (1) analysing; (2) evaluating; and (3) creating, while the other three levels are categorised as lower-order thinking. The conception of higher-order thinking was then developed by various researchers more subtly by defining higher-order thinking as a level higher than simply remembering the existing facts or doing something as students' have done before according to the examples. Higher-order thinking skills is related to working in complex situations, thinking with non-algorithms, solving problems that cannot be predicted and produce many answers (Thomas & Thorne, 2014; Stein & Lane, 1996; Senk, Beckmann, & Thompson, 1997; Weiss, 2003; Resnick, 1987, Thompson, 2008).

Developing higher order thinking is important for several reasons: (1) to organise knowledge learned into long term memory; (2) to develop adaptability in view of a variety of new problems found in life which could become the exercises to develop a higher order thinking ability in formal education (this would eventually develop an attitude and a way of creative thinking to get out of life problems, which are complex); and (3) to encourage the creation of quality human resources that can compete with other nations (Samo, Darhim, Kartasasmita, 2017). The last point shows the broader and long-term idea which has become the focus of 21st century learning. This is a need that does not need to be debated anymore so it indeed requires real practice to prepare our students to deal with it.

The importance of developing higher-order thinking requires a complete strategy that can be applied systematically to access it. King, Goodson, Rohani (2011) reveals that a major factor in the growth of higher order thinking skills is a student-centred classroom. It supports the open expression of ideas, provides active modelling of thinking processes, develops thinking skills, and motivates students to learn. Without it, students will not persist in higher level thinking processes. In this open environment, a teacher’s awareness of students' motivation can dramatically affect the students' progress. Furthermore, Kruger (2013) explains how we can encourage higher-order thinking in our students: Look for their strengths! Observe your students, focus on who demonstrates strengths in the components of higher-order thinking (creativity, problem solving, critical thinking, logical thinking, mental representation, etc.). It is essential to understand how emotions can affect brain function and students who are upset or afraid or stressed, will have a more difficult time learning. It is also important to appreciate that in order to create more opportunities for “insight”, the teachers need to allow the students’ minds to become ‘quiet’. Yee (2002) elaborates that to gauge whether higher-order thinking is
happening in a mathematics classroom, there are three characteristics that can be observed: (1) you can hear students explaining, conjecturing, describing patterns or communicating their ideas (There is direct teaching of specific problem solving and reasoning strategies), (2) you can hear teachers asking students why, what and how-questions that demand more than one-word answers (There is an emphasis by the teacher on meaning and understanding), (3) you can observe students making choices about what procedure to use, or how to integrate knowledge to new and non-routine tasks, monitoring progress and evaluating solution (There is classroom atmosphere that encourages student autonomy, persistence and independent thinking). Collins (2014) reveals that the teachers can do this through the following stages: (1) specifically teaching the language and concepts of higher-order thinking, (2) planning classroom questioning and discussion time to tap into particular higher-order thinking skills, (3) explicitly teaching subject concepts, (4) providing scaffolding, (5) consciously teach to encourage higher order thinking.

Some of the strategies above are the strategies of developing higher-order thinking in classroom learning activities. The development of higher-order thinking skills in terms of learning activities should be concerned with student-centred learning, building motivation, investigating students' thinking strengths and evaluating learning content with questions about higher-order thinking levels. Each expert discloses strategies according to studies, experience and personal interpretation. Each of them can be read and complete each other. So, what are the strategies that can be revealed in different life situations, under the conditions of different students' skills and cultural influences that are still inherent in the students? This study tries to explore the possibility of developing broader higher-order thinking so that students and readers are able to optimally develop themselves and their higher-order thinking skills.

**Materials and Method**

This research uses qualitative methods with a grounded theory approach. The grounded theory (GT), which later in the research method is called grounded research (GR), is a systematic qualitative procedure used to produce theories that explain a process, action or interaction of a substantive, in broader conceptual level, (Creswell, 2015). The research subjects were selected by theoretical sampling techniques using a procedure that involves simultaneous and sequential data collection and analysis (Creswell, 2015). The subjects selected were mathematics education students, in the Faculty of Teacher Training and Education, Nusa Cendana University, Indonesia. The data in this study was descriptiveand was obtained by researchers from written test activities, interviews and literature studies. Interviews were conducted to explore all the subjects' activities in their daily environment, at home, within their culture, as well as their learning experience or other related aspects that can be a source of data in theory formation. The data analysis uses three stages:open coding, axial coding and selective coding.
The research diagram is presented in figure 1:

**Figure 1. Research Procedure**

![Research Procedure Diagram](image)

Figure 1 shows the research procedure. The first stage is subject selection, higher-order thinking skills test and interview. Higher-order thinking skills test results are used to categorise and provide an overview of the subject's abilities. The difference in the subject category is used as the main information for investigating the influencing factors. The interview process explores the supporting factors of HOTS ability comprehensively. The results of the interview were analysed with the coding process to get a category and subcategory and a narrative relationship between categories, propositions, and finally an empirical theory.

**Result and Discussion**

**Analysis of Categories / Themes**

In this process, the researcher examines and analyses the results of the higher-order thinking skills test. Based on the results of documentation analysis, interviews were then conducted related to the writing of higher-order thinking skills to classify the subjects in the category of higher-order thinking skills based on the mathematical problem solving skills, mathematical reasoning, mathematical critical thinking, and mathematical creativity. Furthermore, in-depth interviews about the subjects' thinking habits, influencing factors and other things relevant to the development of the subjects' higher-order thinking skills were conducted. Open coding is done by giving a sign (number) in each subjects' answers related to the development of higher-order thinking skills. This open coding eventually provides six categories with individual subcategories that influence or determine the development of higher-order thinking skills. The six categories and the sub-categories are 1) the cultural influences of the people in East Nusa Tenggara (NTT), Indonesia, 2) the perceptions of mathematics, 3) individual skills, 4) socio-economic support, 5) self-regulation, and 6) learning experiences. The following is an open coding process for the formation of categories about mathematics and cultural influences and their subcategories based on the results tests of higher-order thinking, observation, and in-depth interviews.
In figure 2, the researcher compiles keywords based on the results of in-depth interviews. These keywords are grouped into several groups of keywords as the initial stage of creating categories. After the categories are formed, each category is divided into several subcategories. The same process was carried out to produce the other six categories.

The Findings and Theories that Emerged

Here, the axial process begins with selecting one category from the categories obtained by the Open Coding process and then placed in the middle of the process that is being explored and then linked it to the other categories. The process of selective coding according to Strauss & Corbin (1998) is the process of writing theories that are interconnected among categories in an axial coding model. At the basic level, this theory provides an abstract explanation for the process being examined in this research. This integration process is like writing a storyline that
connects each category. The theory developed can be presented in three ways, specifically: as a visual coding paradigm, as a series of propositions (or hypotheses) or as a narrative story (Creswell, 2015). The general design of the conceptual model for developing mathematical higher-order thinking skills for the students of mathematics education, Faculty of Teacher Training and Education, Nusa Cendana University, Indonesia is presented in Figure 3.

Figure 3. Conceptual models of developing higher order thinking skills

Figure 3 is a conceptual model in the selective coding that shows the influence of each category to the main category, and the relation among each category. The arrows in the conceptual model above show the influence among each category. This influence is based on the literature review of various previous studies, the findings of empirical data of the research subjects and the reflection of the researchers in accordance with the field conditions experienced by researchers. Based on that particular relation a theoretical formulation of the development of higher-order thinking skills is obtained, namely building motivation and belief, contextual learning (cultural culture context), the habit of asking scientific questions, group discussion, higher-order thinking exercises, learning to modify problems, routine discussions in the family and community. These seven points are formed based on the influence of each category with the main category. The model also examines the relation of several categories that influence the strategy that develops mathematical higher-order thinking skills that have not been able to be explained by previous research or existing theories. Therefore, further evidence is needed. The combination of several categories (not all categories) brings out the influences to the development strategies that are not intact, which are thought to affect mathematical higher-order thinking skills.
The First Category is the Culture of the People in East Nusa Tenggara (NTT)

This category consists of oral speech culture, attitude/character, parents' ways to educate and how gender differences in general affect the development of students' mathematical higher-order thinking skills. The four sub-indicators in this category play a major role in the formation of the first strategy, which is to build motivation and confidence and the ability to ask scientific questions. This is in accordance with the opinion of Ginsberg (2015) who states that students' cultural experiences significantly impact how they respond in the classroom. Classes tend to be more effective in developing the capacity of the students which come from various backgrounds if the teacher understands how the culture can shape learning and how teachers can develop intrinsic motivation of the students with diverse cultures in the classroom. In line with Ginsberg, Trumbull & Rothstein-Fisch (2011) reveals that the achievement of motivation is something that everyone wants to support the students, but some may realise that this is influenced by the culture. The meaning of “achievement” is the cultural variable and the motivation that the students hold. Therefore, how to achieve it might be different from one to another, depending on their cultural background. Furthermore, Igbo, Sam, Onu & Dan (2015) reveal that gender differences have a significant effect on learning motivation at school. Male students have a higher learning motivation than their female classmates. This is not a surprising result since gender differences in student motivation might be a result of the influence of parents' low expectations of the girls' education. Parents tend to believe that investment in the boys' education is more important than that of girls. Mostly, female students are preoccupied by housework including raising children. The differences in parental and community provisions have achieved far-reaching consequences and can explain the gender differences in the students' learning motivation (Innabi, 2003). Furthermore, culture also plays a role in forming scientific questioning strategies. Dkeidek, Mamlok-Naaman & Hofstein (2010) utter that culture, tradition, norms, social structure, lifestyle, and related factors play an important role in developing students' questioning ability and it seems that every effort intended to target questioning skills must be done by shifting the existed paradigm, which concerns the multicultural context to be implemented. Here, it is clear that the cultural or ethnic background of students who are multicultural influences the students' questioning ability.

The Second Category is Individual Skills

This category consists of some subcategories namely, problem solving ability, reasoning ability, critical thinking and creative thinking skills, which are all categorised into one individual skills category. Subjectively, it can be said that individual skills will greatly affect the ability of the students to ask scientific questions and learn to modify questions. A good individual skill will provide a high curiosity impulse that can be realised by exploring more of his/her curiosity by asking questions and modifying the problems faced. The encouragement of curiosity can be a reason to get new information or to strengthen the personal opinions that
The Third Category is Socio-Economic Support

This category consists of the subcategories of parental support, economic support, parents' education and interaction with others. These four sub-categories influence the development of the subjects' higher-order thinking skills as explained in theme analysis 3. These four sub-categories contribute to the strategy of developing higher-order thinking skills, namely building motivation, asking questions scientifically, group discussion and family discussion routines. When the subjects get good support from the family, especially in learning activities, the subjects' readiness in facing learning activities with all the challenges is available. The subjects will come with confidence and good motivation because of the support that he/she gets. Parental supports for learning at home is significantly and positively related to the students' academic motivation and their perseverance in learning mathematics in the classroom which, in the long term, can improve their mathematics achievement. Moreover Igbo, Sam, Onu & Dan, (2015) mentions that a good relationship between parents and children provides a good influence on learning motivation in school. Parent-child relationships involve providing supportive learning resources such as books, newspapers, magazines, and encouragement for the children to read, do and send their assignment on time. Willingness to spare their time to help the children learn at home, care, comfort and affection, adequate arrangements for the children's food, protection, rest, clothing and shelter, active participation in parent-school contacts are all needed to motivate their children to perform better at school. That means, when parents provide their children with nurturing and various stimulation through all areas of social life, academic achievement is significantly increased (Innabi, 2003). When parents can be involved as a resource for academic activities at home, the bridge between the school and the home environment becomes clearer. Thus, children feel more capable to master academic activities in school. Parents can help children in leaning the new material by scaffolding the new activity (Gonzalez-DeHass, Willems, & Holbein, 2005). Having regular discussions at home is a form of good support from parents to equip children with the capacity to ask questions and make decisions. This is related to debriefing for the children to be able to interact in group discussions in the classroom learning activities.

The Fourth Category Is Self-Regulated Learning

This category is related to how the subject regulates him/herself, controls him/herself, regulates learning, motivates him/herself, fosters confidence, regulates anxiety and overcomes learning obstacles. These attitudes will greatly affect the success of the subject in the activities of increasing higher-order thinking skills. Self-regulation is related to the motivational building strategy (Pintrich & De Groot, 1990). Self-regulation comprises several attributes in itself as
part of the strategy of developing the students' higher-order thinking skills while on the other hand influences other strategies such as asking questions scientifically, group discussion, higher-order thinking exercises and learning to modify questions. All the strategies mentioned above will be the right strategies or are able to become a contributing strategy to the development of higher-order thinking ability if it is based on a good self-confidence, good motivation and is supported by other self-regulated attributes.

The Fifth Category Is the Subjects' Views of Mathematics

In the context of the subjects of this research, the views on mathematics in general show similarities that mathematics is abstract; it is the science of logical thinking, and is complicated and challenging. This view in fact does not discourage the motivation of the subjects to learn mathematics. Attitudes toward mathematics have a positive relation with motivation (Mata, Monteiro, & Peixoto, 2012). The view of mathematics as the ability to think presents different things. Subjects who are accustomed to mathematics as a matter of quantification change their conceptions of mathematics in forging thinking skills. A positive changing of view enables the subjects to survive and be ready to face mathematics in the form of any thinking review. This condition contributes to motivation and confidence in the strategy of developing higher-order thinking skills.

The Sixth Category Is Learning Experience

Based on the previous theme analysis, learning experiences both at the previous education level and personal learning experience ready the subjects to face learning activities which in this case relates to the strategies to build motivation and confidence, ask scientific questions and learn to modify questions. Learning experiences in various opportunities are used by the teachers as a reference in the preparation of learning tools (Hailikari, Katajavuori, & Lindblom-Ylanne, 2008; Pritchard, Lee & Bao, 2008). The exploration of learning experiences aims to produce learning designs that activate or use learning experiences as encouragement or support in increasing the students' participation and abilities. Students who come with a good learning experience background must be used to increase their capacity to use existing experience to learn something new. Students who come with a minimal learning experience background must be supported more intensively to learn something new. Both types of students have different experiences and different types of support in learning.

Main Category: Development of Higher Order Thinking Skills

Building Motivation and Confidence

Some students suggest developing higher-order thinking skills first by convincing themselves that they are able to deal with these problems. Building confidence is one of the biggest
motivators in being able to deal with any situation. Students believe that laziness and doubt are obstacles to learning. Other students don't mention the same thing but in various theories, motivation is fundamental to being able to do anything. Motivation can be built from the inside like the belief that being able to think at a high level but also from outside is the desire to be able to make parents proud. Building motivation and confidence is placed as the first strategy in developing higher-order thinking abilities for students because motivation and confidence are the basis for further work.

**Contextual Learning**

Students express that the use of contextual problems in learning could strongly support them to think and imagine. Learning with real or contextual problems can improve their thinking skills because the focus of learning is clear, has benefits, and the dynamic situation of the problem can encourage them to be more adaptive. This is in line with components in contextual learning, namely critical thinking analysing and achieving high standards. Contextual learning in accordance with this study has had a positive impact on the development of higher-order thinking abilities of students compared to ordinary learning. Contextual learning provides the possibility for students to experience learning as a daily activity in which there is the connection between material content and life. Furthermore, students also have the opportunities to discuss and express ideas with friends and to deal with problems that are not routine.

**Habit of Asking Scientific Questions**

Some students expressed the strategy to develop higher order thinking skills by asking questions that are critical and analytical. Researchers categorise this type of asking as a type of scientific questioning that is the type of inquiry based on fact. The interview results show that students realise that scientific inquiry is an encouragement to think differently or from within another context. Students have experienced learning with the researchers who use presumptive and reasoned questions, thus helping the subject to think differently.

**Group Discussion**

All students expressed that group discussion helped them to better understand the problem and achieve a better level of their thinking. The interview results show that students realise that group learning helps them find or get something new. By studying in groups each person can exchange ideas, equalise perceptions and correct them by asking critical questions to the discussion partners. Group discussion will also encourage students to study independently as well.
Hots Exercises

All students revealed that the way to train and develop higher-order thinking skills was by continuing to practice higher-order thinking type problems both in the classroom and in personal learning activities. The interview results showed that students revealed the plan based on the understanding that being familiar with mathematical problem solving would train the brain to think. The provision of higher-order thinking questions in learning activities teach them to think and work out of context.

Learning to Modify Problems

An interesting thing expressed by the students in developing higher-order thinking skills is the modification of the questions. Modifying the understanding of students is to practice making new problems from existing problems. Modifying the questions then solving them has provided very good input for students in supporting students' thinking abilities. Students benefit from that habit. Modifying the problem can be done by asking yourself what if the problem is changed with different facts, why the facts are given like this, what if the facts are changed, why use this concept in solving it.

Routine Discussions in the Family and Community

Some students stated that the habit of discussion in family and community is one way to develop higher-order thinking skills. The habit of discussing in the family supported someone's ability to express their opinions, make decisions and take responsibility in the decisions taken. The habit of discussion in this family becomes an important task for parents, especially in directing and shaping the way children think. Parents should get children to talk with discussions that build and implement ideas.

Conceptual Models between Categories

Several categories influence each other which shows their integration in developing students’ higher-order thinking skills. The integration of several categories can in fact eliminate some of the verbal strategies in the conceptual model of developing higher-order thinking abilities of students but that may not necessarily fail in their grand goals. The process of integrating several categories depends on the influence of these categories and their effects must be proven to develop the higher-order thinking ability of students which will be presented in the form of hypotheses as a conceptual model of propositions. The arrow in Figure 2 shows the effect between each category. Double arrows are a relationship model or a two-way influence, while a single arrow is a one-way influence model. Category 1 has an effect on all categories but on
the other hand it has no effect on category 1. All categories have an effect on the main category but not on all subcategories in the main category.

**Conceptual Proposition Models**

The conceptual model of developing students' higher-order thinking influences each category toward the main category as well as supports each category toward the existing development strategy. In addition, there is a relation between each category and the other categories supported by the theories or results of previous studies. The model also manifests the relation of several categories that influence the development of mathematical higher-order thinking skills that have not been able to be explained by previous research or existing theories. Therefore, further evidence is needed. The combination of several categories (not all categories) brings out the influences to the development strategies that are not intact, and are thought to affect mathematical higher-order thinking skills.

**Self-Regulated Learning Influences the Ability to Ask HOTS Exercises and Learn to Modify Questions**

In general, self-regulated learning has an important role in developing mathematical skills. Good self-regulation could control anxiety and also could strongly support the achievement of good mathematical skills. Self-regulated learning in the conceptual model of the strategy of developing higher-order thinking skills and the literature review influence the strategies of building motivation and confidence. The limited role of SRL raises a new suspicion of the role of self-regulated learning in the ability to ask questions (question-asking ability). This role can be cited since self-regulated learning is related to good self-regulation which raises the curiosity that first arises by asking questions. Other SRL developments can also influence higher-order thinking and the learning problems modification strategies.

**Self-Regulated Learning Affects the Perception of Mathematics**

The perception about mathematics is mostly formed by external support, which is related to the culture and views of parents and the surrounding environment. Some subjects express that their beliefs and motivations affect their perceptions of mathematics. The presence of external support raises allegations that internal support is possible with the development of SRL.

**Different Cultural Backgrounds as well as Socio-Economic Support Influence the Strategy of Developing Higher-Order Thinking Ability**

If we examine the influence of these two categories partially, each of them will support the strategy of building motivation and confidence and scientific questioning, while socio-
economic support forms a scientific questioning strategy, group discussion and family discussion routine. This relationship has degraded several other strategies which are higher-order thinking and learning to modify questions. From the interviews, information was obtained that cultural background affects all existing categories. Moreover, socio-economic support affects all other categories except the cultural categories of the NTT community. Therefore, the cultural background and economic support should influence all strategies in developing higher order thinking skills. Its influence toward the strategies needs to be further proven.

**Perception of Mathematics and Experience Jointly Influencing the Strategy of Developing Higher-Order Thinking Skills**

Each of them contributes to the influence of building motivation and beliefs as well as habit of asking scientific question and learning to modify questions. This support seems to have a considerable effect which can be evaluated with the support of the six categories simultaneously.

**Conclusion**

The empirical theoretical formulation of the development of higher-order thinking skills in mathematics education students is the development of higher-order thinking skills that can not only be achieved by classroom learning strategies but also a comprehensive conceptual design involving the personal elements of students, parents, the environment and classroom learning. Development of higher-order thinking skills includes building motivation and confidence, contextual learning (culture-based), the habit of asking scientific questions, group discussion, higher-order thinking type exercises, learning to modify questions, and routine discussions in the family and community. The recommendations for further studies is to validate the above empirical theory quantitatively using statistical tests and quantitative tests for conceptual proposition models.

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