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Analysis of Newly Proposed Efficient Experimental Designs Strongly Balanced for Carryover Effects when $p<v$

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Repeated measurements designs (RMDs) special kinds of experimental design have gained much attention today in the field of experimentation. Due to the flexibility in different constraints, these designs have great practicality in a variety of fields especially in dairy farms and veterinary experiments. But there is one problem that arises in such experimentations which are called Carry-over effects. Such effects can be balanced out using Balanced RMDs (BRMDSs). But these designs do not give equal efficiencies for both treatment and carry-over effects which is the goal of every experimentation. In such situations, Circular Strongly balanced RMDs (CSBRMDs) have been suggested. The present paper focuses on the analysis of sheep weight data (lb) by using efficient CSBRMDs proposed by Daniyal et al, (2020) through a well know method of cyclic shifts. Designs obtained by this construction independently estimate direct and carry-over effects with equal efficiencies and hence they can be recommended as compared to any other designs. Discussion related to contrasts has also been added and a list of designs for future experimentation has also been mentioned. This study of ANOVA facilitates the experimenters to use these efficient designs when the number of treatments is odd and p periods $<v$ treatments. The list of designs using the procedure proposed by Daniyal et al, (2020) can be used for large experiments in dairy form as well as in other experimental studies. Experimenters will have more flexibility in the choice of

designs in terms of treatments, period sizes, and experimental subjects.

Keywords: *Carry-over effects; Balanced repeated measurements designs; Strongly balanced repeated measurements designs; ANOVA.*

1. Introduction

A repeated measurement method (RMD) is an experiment in which a control unit, or short sample, is consistently subjected to a series of treatments (Chasiotis & Kounias, 2019). When a pre-period is added, an RMD is called circular (CRMD). For each pre-term unit, the procedure is the same as for the last term (Chasiotis & Kounias, 2019). In the early 1930s, the basics of the statistical approach towards experimentations were established by R. A. Fisher. Design of experiments was firstly used in agricultural experimentation but due to different flexibility in constraints, it has applications in almost every field of sciences and experimentations (Daniyal et al., 2020). The design of experiments purpose is to make a comparison between v treatments based on responses produced by experimental subjects. Considering the example from agriculture, the treatments can be varieties of wheat or different seeds, in engineering, these can be levels of pressure or heat, and considering chemistry experiment, the purpose can be to make compare different chemicals used in medicine. The constraints of the experiment demand a variety of designs and their analysis (Hussain, et.al, 2019). In an RMD every experimental subject gets a sequence of v treatments in successive p periods. Such designs can be considered as the row-column design consisting of a set of experimental units/subjects expressed across columns and a set of periods (of time) presented across the rows wherein the experimental subject receives few or complete set of treatments, one at a time, over the periods. The nature of such an experiment is that any treatment applied to a subject in a certain period creates influence on the response of the unit not only in the current period of its application but also in the successive periods. It means that the treatment leaves a carry-over effect in the periods after its period of direct application. The measurement taken on the unit depends upon treatment managed in the current period called direct effect, the treatment in the previous period (first-order carry-over effect) (Jankar, Mandal & Yang, 2020).Among many designs which are available for treatment comparison experiments, the repeated measurements designs have occupied an important place. Buta problem arises when there are carry-over effects in the experimentation which must be balanced out. It can be balanced out by using a washed-out period but such periods are lengthy, time-consuming, and expensive and therefore not recommended in most situations. Balancing the carry-over effects is suggested in such cases. However, the Balanced RMDs do not provide the estimates of direct and residual effects of treatments independently and therefore provide estimates of residual effects with less precision as compared to direct effects. Designs that provide estimates of direct and residual effect independently with equal precision are CSBRMDs and therefore preferable over

BRMDS. These designs are likely to be useful when increased precision is required in the estimation of residual and treatments effects.

The present article focuses on the analysis of circular strongly balanced RMD considering the data of sheep weights in pounds. The method of construction of this RMD has been proposed by Daniyal et al, (2020) in which an efficient combination of *v* and *p* has been used. These designs give equal efficiency for both the carry-over and direct effects making it more suitable because these effects are replicated the same number of times in the experimental study. This increases the reliability of the experiment. The construction method together with the analysis provides the experimenters an easy way to get reliable results.

2. Methodology

Model for Strongly BRMDs when $p<v$

The following response model has been assumed:

$$Y = \mu E + D\delta + R\rho + Uv + P\pi + e \qquad \textbf{(2.1)}$$

where **Y** = vector with observations of order $np \times 1$, $\boldsymbol{\mu}$ = overall mean, $\boldsymbol{\delta}$ is $v \times 1$ vector of direct effects, $\boldsymbol{\rho} = v \times 1$ vector of carry-over effect, $\mathbf{v}$ is $n \times 1$ unit vector , $\boldsymbol{\pi} = p \times 1$ period effects vector of order and $\mathbf{e} = np \times 1$ column vector of random error having mean zero and constant variance σ^2. ***D*** = incidence matrix of direct effect. ***R*** = incidence matrix of carry-over effect. ***U*** = $np \times bv$ unit effect incidence matrix. ***P*** = period incidence matrix. According to proposed method such designs can be constructed with $p = r$, for $v = ri$; i odd with $\lambda' = 1$ through the i sets of shifts. $S_j = [l_{j1}, l_{j2}, \ldots, l_{j(r-1)}]$. with the conditions **(i)** $0 \le l_{j1}, l_{j2}, \ldots, l_{j(r-1)} \le v-1$, **(ii)** S^* contains each of $0, 1, 2, \ldots, v-1$ exactly once **(iii)** $S^* = [l_{j1}, l_{j2}, \ldots, l_{j(r-1)}, v-\{(l_{j1}+l_{j2}+ \ldots + l_{j(r-1)}) \bmod v\}]$ Considering this theorem, CSBRMD with $v = 9$ with the sets of shifts [1,8]+[2,3]+[5,6] has been obtained through MCS (Rule I). Using this methodology, how a design is constructed through given set of shifts is illustrated here for $v = 9$, $p = 3$, $S_1 = [1,8]$, $S_2 = [2,3]$ and $S_3 = [5,6]$. Here $S^* = [0,1,2,3,4,5,6,7,8]$

Step 1: Consider v experimental subjects for $S_1 = [1,8]$. Write 0 to v-1 in each subject of first period. Add 1 mod (v) to every value of first period of each subject to obtain every subject of second period. Then add 8 to every value of second period for each subject to obtain values of third period.

Step 2: Consider v more experimental subjects for $S_2 = [2,3]$. Write 0 to v-1 in each subject of first period. Add 2 to every first period value of each subject to obtain values of second period. Then add 3 to second period value of each subject to obtain values of third period.

Step 3: Consider another *v* subjects for S_3 = [5,6]. Write 0 to *v*-1 in each subject of first period. For second period of every subject, add 5 to every value of first period of each subject. Then add 6 to every value of second period for each subject of third period.

Periods	**Experimental Subjects**								
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1	0	1	2	3	4	5	6	7	8
2	1	2	3	4	5	6	7	8	0
3	0	1	2	3	4	5	6	7	8

Periods	**Experimental Subjects**								
	10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
1	0	1	2	3	4	5	6	7	8
2	2	3	4	5	6	7	8	0	1
3	5	6	7	8	0	1	2	3	4

Periods	**Experimental Subjects**								
	19	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**
1	0	1	2	3	4	5	6	7	8
2	5	6	7	8	0	1	2	3	4
3	2	3	4	5	6	7	8	0	1

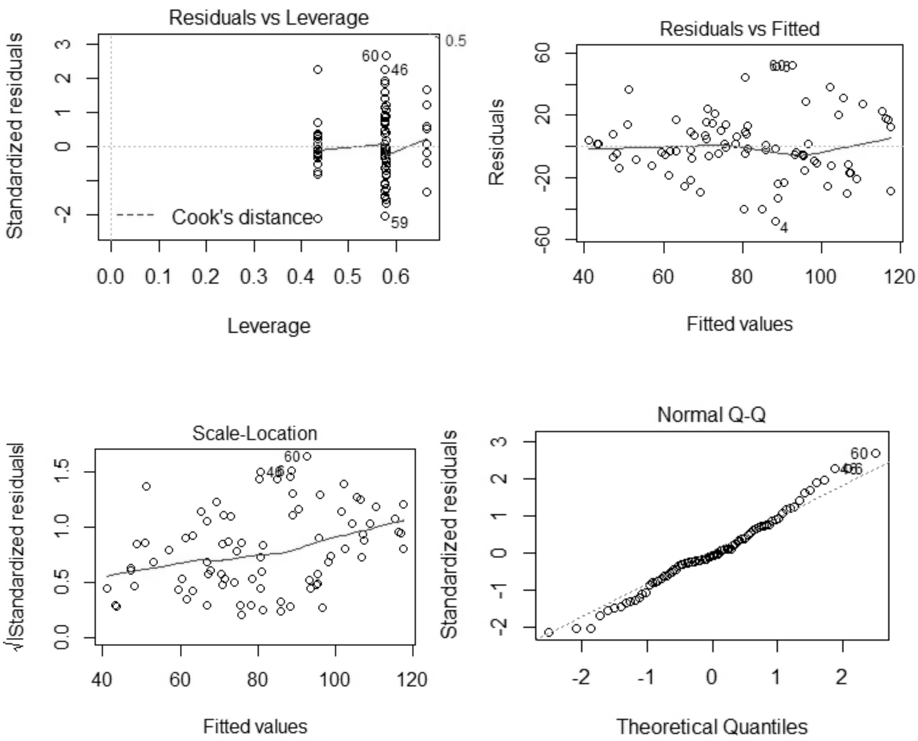
By combining the above three steps, a complete design can be obtained. The newly proposed Circular Strongly BRMDS has been used at the local sheep breeding farm on 27 crosses bred sheep (Experimental Subjects) with the age of almost one year to make a comparison between the effects of nine ingredients on the growth of the sheep expressed in table 2.1. Nine ingredients (g/kg) were given over 3 periods i.e. $p = 3$ with a duration of 4 months each. The weights of the sheep were records (lb) at the end of each period. Treatments/ ingredient (g/kg) used can be 0 = Barley, 1 = Dried grass, 2 = Dried Lucerne, 3 = Oat feed, 4 = Citrus pulp, 5 = Sugar, 6 = beet pulp, 7 = Soya-bean meal, 8 = Fish meal.

Table 2.1: Sheep weight data (lbs) and calculations

	SHEEP WEIGHT									
SHEEP NUMBER	**1**	**2**	**3**	**3**	**4**	**5**	**6**	**7**	**8**	**SUM**
Periods **1** **2** **3**	(0) 80 (1) 45 (0) 85	(1) 40 (2) 60 (1) 140	(2) 65 (3) 90 (2) 60	(3) 95 (4)125 (3) 90	(4) 43 (5) 94 (4) 59	(5) 78 (6) 67 (5) 87	(6) 90 (7) 85 (6) 90	(7) 76 (8) 98 (7) 65	(8) 76 (0) 90 (8) 89	643 754 765
Sum	210	240	215	310	196	232	265	239	255	**2162**
SHEEP NUMBER	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**SUM**
Periods **1** **2** **3**	(0) 89 (2) 87 (5) 55	(1) 87 (3) 75 (6) 35	(2) 87 (4) 45 (7) 125	(3) 88 (5) 65 (8) 89	(4) 80 (6) 75 (0) 65	(5) 45 (7) 55 (1) 45	(6) 125 (8) 89 (2) 80	(7) 56 (0) 90 (3) 88	(8) 69 (1) 76 (4) 44	726 657 626
Sum	231	197	257	242	220	145	294	234	189	**2009**
SHEEP NUMBER	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**SUM**
Periods **1** **2** **3**	(0) 140 (5) 88 (2) 90	(1) 45 (6) 40 (3) 145	(2) 138 (7) 76 (4) 84	(3) 40 (8) 88 (5) 40	(4) 45 (0) 40 (6) 95	(5) 45 (1) 138 (7) 134	(6) 130 (2) 134 (8) 76	(7) 90 (3) 95 (0) 80	(8) 137 (4) 77 (1) 56	810 776 800
Sum	318	230	298	168	180	317	340	265	270	**2386**

Fig 2.1 are the graphs to verify the assumptions of model 2.1. Plots show influential cases. The Scale-Location plot shows whether our residuals are gathered equally along with the predictor range, i.e. Homoscedastic.. It can be observed that points are randomly plotted around the line. The line is above the residuals for those predictor values are more spread out. This data generally has uniform variance at ends of our predictor range and is Heteroscedastic (i.e. non-uniform. variance) in the middle of the range. The normal Q-Q plot shows that data is normally distributed. Our plot looks quite good and indicates normal distribution, as it's generally in a straight line.

Fig 2.1. Graphs to Verify Assumptions of Repeated Measurements Designs



Necessary calculations are as below;

Treatment	Sum of observations on treatments D_h	Sum of observations following treatments R_h	Sum of the total for sheep receiving treatment Last L_h	W-values W_h
0	759	742	695	438
1	672	660	655	-392
2	801	812	827	726
3	806	649	774	829
4	602	730	683	-1120
5	597	705	631	-1118
6	747	827	642	371
7	762	710	813	350
8	811	754	837	816
	6557	**6589**	**6557**	**900**

Treatments	Direct Effect	Carry-over Effect	Permanent Effect
0	759	9.989	769
1	672	-72.1111	600
2	801	79.889	881
3	806	-83.1111	723
4	602	-2.1111	600
5	597	-27.1111	570
6	747	94.8889	842
7	762	-22.1111	740
8	811	21.8889	833
Mean	**729**	**0**	**729**

Analysis of Variance and the calculations proceed as follow:

Correction Factor $= \frac{6557^2}{81} =$ 530793.19,

Corrected Total sum of squares $= \{(80)^2 + (45)^2 + \ldots + (56)^2\} - 530793.19 =$ 64813.81,

Periods sum of squares: $\frac{1}{27}\{(2179^2 + 2187^2 + 2191^2)\} - 530793.19 = 2.80$

Sheep weights sum of squares: $\frac{1}{3}\{(210^2 + 240^2 + \cdots + 270^2)\} - 530793.19 = 20681.11$

Direct effects sum of squares: $\frac{1}{(9)}\{(759^2 + 672^2 + \cdots + 811^2)\} - 530793.19 = 5517.88$

Direct Effect (Adjusted) $= 6327.81 - \frac{65604}{81} = 5517.884$

Carry-over sum of squares: $\frac{1}{(9)}\{(742^2 + (660)^2 + \cdots + 754^2)\} - 530793.19 = 8448.91$

The analysis of variance can also be carried out for the sheep weights using SPSS as below; One way strongly balanced repeated measurements designs in SPSS can be carried out by steps “Analyse” menu > “General Linear Model” > “Univariate”.Table 2.2 shows some entries of data in SPSS and table 2.3 is the final ANOVA table obtained. The data entered in SPSS for analyzing RMDs is different from other ANOVA models because, in its direct effect, carryover effect, and period effects have been considered. The syntax of SPSS used for the analysis is;

```
UNIANOVA sheep weight BY units period treatment carryover
 /METHOD=SSTYPE (3)
 /INTERCEPT=INCLUDE
 /CRITERIA=ALPHA(0.05)
 /DESIGN=unit period carryover treatment.
```

Table 2.2: Format of entering Data in SPSS

unit	period	direct	carryover	Sheep weight
1	1	0	0	80
1	2	1	1	45
1	3	0	0	85
2	1	1	1	40
2	2	2	2	60
2	3	1	1	140
3	1	2	2	65

Table 2.3: ANOVA table along with the calculations

Source of Variation	Degree of Freedom	SS	MSE	F-Statistic
intercept	01	530793.19		
Sheep weights	26	20681.11	795.42	0.95
period	2	2.80	1.4	0.001
Direct (adjusted)	8	5517.88	689.73	0.82
carry-over	8	8448.91	1056.11	1.26
Error	36	30,163	837.86	
total	81	595607.00		
Corrected total	**80**	**64813.81**		

The experimental designs proposed and applied on the data are constructed and analysed on the assumption that carry-over effects of treatment continue to the next period. Williams (1949, 1950) has proposed the designs which allow the estimation of carry-over effects that last more than one period, particularly into two periods only. Similarly, Patterson (1952) has published a series of incomplete Latin-squares(fewer periods than treatments) that are balanced for one-period carry-over effects only. In the proposed series of designs in literature, the precision with which carryover effects are estimated is considerably less than direct effects. This is because of the reason that carryover effects are replicated less than the direct effects. But in our proposed structure of designs, both the effects are replicated an equal number of times as mentioned in table 2.3, degree of freedom for both effects is 8. This gives equal efficiencies to direct and carryover effects.

Considering the efficiency of this design, the Following are the identities under model 2.1.

$D'D = bpI_v$,$D'R = L$,$D'U = N$,$D'P = bE_{v,p}$,,$R'U = M$,$U'U = pI_n$,$U'P = EP'P = nI_n$, where $E_{k,q}$ is a matrix of order $k \times q$ having entries one and I_q is the identity matrix of order $q \times q$, the reduced normal equations for $\hat{\delta}$ and $\hat{\rho}$ will be:

$$C\begin{bmatrix}\hat{\delta}\\ \hat{\rho}\end{bmatrix} = \begin{bmatrix} bpI_v - p^{-1}NN' & L' - p^{-1}NN' \\ L' - p^{-1}NN' & bpI_v - p^{-1}NN' \end{bmatrix}\begin{bmatrix}\hat{\delta}\\ \hat{\rho}\end{bmatrix} = \begin{bmatrix} D'Y - p^{-1}NU'Y \\ R'Y - p^{-1}NU'Y \end{bmatrix}$$

$\mathbf{\theta} = bpI_v - p^{-1}NN', \mathbf{\Theta} = bpI_v - p^{-1}NN'$

$\mathbf{\theta}$ = [5.33 - 0.6667 - 0.6667 - 0.6667 - 0.6667 - 0.6667 - 0.6667 -0.6667 - 0.6667]

$\mathbf{\Theta}$ = [-2.667 0.333 0.333 0.333 0.333 0.333 0.333 0.333 0.333], **L** = [1 1 1 1 1 1 1 1 1],

The information matrices of direct and carry-over matrices are redundant and can be expressed with their initial rows. From analysing the information matrices direct and carry-over the row sum of information matrices are zero and diagonal entries of matrices are the same and off-diagonal elements are the same in both information matrices i.e. the matrices of both direct and carry-over effects are symmetric. The eigenvalues of the information matrix are known as canonical efficiency factors (James & Wilkinson 1971; Pearce, Calinski & Marshall 1974). The efficiency for both direct and carry-over effects can be computed as the harmonic mean of non-zero eigenvalues of their information matrix. Both the effects share the same efficiencies as 0.54. As we increase the number of treatments together with period sizes, the efficiency of the designs is increased as illustrated in table 4.1.

3. Contrast Analysis of Treatments

In many experiments, one may desire to partition the sum of treatments sum of squares into several components based on one degree of freedom each. Let Q_j be any contrast among the treatments T's. Then the sum of squares for this contrast can be computed as

$Q_j = \frac{(\text{contrast})^2}{r\sum(\text{contrast coefficient})^2} = \frac{Q_j^2}{r\sum c^2}$, where c^2 are the constants coefficients of treatment total in the jth contrast and r is the number of observations in each total. The quantity $\frac{Q_j^2}{r\sum c^2}$ is the component of treatment/direct SS and it has one degree of freedom. Thus, the treatment degree of freedom v - 1 can be further partitioned into v - 1 contrasts returning to our example we can classify our treatments like

Ingredients	Barley	Dried grass	Dried Lucerne	Oat feed	Citrus pulp	sugar	Beet pulp	Soya bean meal	Fish meal
Treatments	T0	T1	T2	T3	T4	T5	T6	T7	T8

One can make the following differences

T1-T2 = effect of dried grass ingredient versus Dried Lucerne, T3-T4 = effect of oat feed versus citrus pulp, T5-T6 = effect of sugar versus beet pulp. A useful way of expressing these differences can be in the following form $\frac{1}{2}\big((T1-T2)+(T3-T4)\big)$,$\frac{1}{2}\big((T3-T4)+(T5-T6)\big)$, $\frac{1}{2}\big((T1-T2)+(T5-T6)\big)$, For convenience, the notations for the contrasts can be given as;**C1** = $\frac{1}{2}(T1-T2+T3-T4)$, **C2** = $\frac{1}{2}(T3-T4+T5-T6)$, **C3** = $\frac{1}{2}(T1-T2+T5-T6)$. In vector form for **C1,** the coefficients can be expressed as (1/2, -1/2, 1/2, -1/2). The sum of these coefficients adds up to zero, here T's are treated as the mean of treatments. Now calculating contrast we have **C1** = 4.165, **C2** = 2.995 and **C3** = -15.5. Testing the hypothesis that **C3** = 0 or not, the variance for **C3** = $\frac{837.8611}{9}$ and the standard deviation is 9.64, t-statistic value = -1.606 with p-value 0.147 showing insignificant results which means that the effect of dried grass versus dried Lucerne is the same as the effect of sugar versus beet pulp.

4. Concluding Remarks

The strongly balanced RMDs have attained great attention nowadays in the field of animal experimentations because of their estimable property of direct and carry-over effects independently. This layout can be extended to several treatments and period sizes. They allow the comparison of treatments on a within-sheep as well as within-period effect. Design in which each treatment is preceded by every other treatment is considered balanced and if each treatment is preceded by every other treatment including itself is considered strongly balanced RMDs. In all published designs in the literature, the efficiency of carry-over effects is less than the direct effects since the carry-over effects are repeated less than the direct effects. But in our proposed structure of design which is strongly circular balancedcarry-over effect is replicated the same number of times as direct effect and therefore share equal efficiency.Table 4.1 is the list of designs using the procedure proposed by Daniyal et al, (2020) which can be used for large experiments in dairy form as well as in other experimental studies. Experimenters will now have more flexibility in the choice of designs in terms of the number of treatments, period sizes, and the number of experimental subjects. When the experimenter is sure about the combination of treatments with period size (s), he has the choice of design from a given catalog. This study facilitates the experimenters to use these designs when the number of treatments is odd and $p<$ v. These designs show the 100% efficiency of separability proposed by Diveche and Gondaliya (2014) which suggested that such layout of the treatments can distinguish the direct effect and carry-over effect separately 100%.

Table 4.1: List of tables for $p<v$ with direct and carry-over efficiencies

v	*p*	Sets of Shifts	The efficiency of Direct Effect	Efficiency of Carry-over Effect	Efficiency of Separability
15	3	[1,14]+[4,8]+[7,2]+[10,11]+[13,5]	0.60	0.60	1.00
15	5	[1,2,3,9]+[8,6,7,4]+[11,12,13,14]	0.72	0.72	1.00
25	5	[1,2,3,19]+[6,7,8,9]+[4,12,13,16]+ [14,17,18,11]+[21,22,23,24]	0.80	0.80	1.00
21	7	[1,2,3,4,5,6]+[10,9,8,12,11,14]+ [15,16,17,18,19,7]	0.88	0.88	1.00
35	7	[1,2,3,11,34,19]+[8,9,10,4,12,20]+ [15,16,28,18,26,13]+ [22,23,14,25,6,29]+[27,30,31,32,33,5]	0.92	0.92	1.00
27	9	[19,2,3,4,5,6,7,8]+ [12,1,11,13,16,17,14]+ [21,20,10,22,23,25,24,26]	0.78	0.78	1.00
45	9	[1,2,3,4,5,6,7,17]+ [10,21,12,13,14,15,16,8]+ [19,29,11,22,23,24,25,9]+ [28,20,30,31,32,33,34,35]+ [37,38,39,40,41,42,43,44]	0.85	0.85	1.00
33	11	[1,2,3,4,5,6,7,8,9,21]+ [12,13,14,15,16,17,18,19,20,10]+ [23,24,25,26,27,28,29,30,31,32]	0.89	0.89	1.00
55	11	[1,2,3,4,5,6,7,8,20,54]+ [12,13,14,15,16,17,18,19,9,21]+ [35,24,25,26,27,28,29,30,31,32]+ [34,23,36,37,38,39,40,41,42,22] [45,46,47,48,49,50,51,52,53,10]	0.92	0.92	1.00
39	13	[1,2,3,4,5,6,7,8,9,10,11,12]+ [27,15,16,17,18,19,20,21,22,23,24,38]+ 26,14,28,29,30,31,32,33,34,35,36,37,25]	0.92	0.92	1.00
65	13	[1,2,3,4,5,6,7,8,9,10,11,64]+ [14,15,16,17,18,19,20,21,22,23,24,25]+ [12,28,29,30,31,32,33,34,35,36,37,40]+ [38,41,42,43,44,45,46,47,48,49,50,53]+ [51,54,55,56,57,58,59,60,61,62,63,27]	0.94	0.94	1.00
45	15	[1,2,3,4,5,6,7,8,39,10,11,12,13,14]+ [16,17,18,19,20,21,22,23,9,25,26,27,28,29]+ [31,32,33,34,35,36,37,38,44,40,41,42,43,24]	0.93	0.93	1.00
75	15	[2,3,4,5,20,7,8,9,10,11,12,59,74]+ [16,17,18,19,6,21,22,23,24,25,26,27,28,13]+ [31,32,33,34,35,36,37,38,39,40,41,42,43,14]+ [47,46,48,49,50,51,52,53,54,55,56,57,58,29]+ [61,62,63,64,65,66,67,68,69,70,71,72,73,44]	0.96	0.96	1.00

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