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The Effect of The Neural Branching Thinking Strategies in Teaching Mathematics on Developing the Mathematical Proficiency for High School Students

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This research aims to identify the effect of neural branching thinking strategies on developing mathematical proficiency among secondary school students. The study sample represented (63) students of the first year of secondary school in the eighth secondary school in Al-Khobar Governorate, Saudi Arabia Kingdom. In order to achieve the objectives of the research, the researchers prepared the test of mathematical proficiency, as well as the productive disposition scale to collect data according to the quasi-experimental approach, and the results showed that the experimental group surpassed the control group in both mathematical proficiency as a whole and each of its dimensions , as well as in the measure of productive disposition as a whole and in the propensity axis towards learning mathematics, and according to this results, it was recommended with the importance of the mathematics teachers activating the strategies of branching thinking in the teaching of mathematics at different learning grades.

Keywords: *Neural Branching Thinking Strategies, Teaching Mathematics, Mathematical Proficiency, High School Students.*

Introduction

Our current era is witnessing rapid development and progress in various areas of life. This constitutes a great challenge for societies to keep pace with these developments, which are a major reason for competition among them. In order to be among the first grades, we must focus on the continuous mechanism of change in the learning and teaching processes. Through these two processes, generations for the future are prepared, characterized by creativity and thinking, so that these generations will be the basis for society to face the challenges of this era. Therefore, students are the focus for investing in human capital within educational institutions.

High school students, in particular, consider this stage to be crucial for their personal life, through which their future is determined. Therefore, attention must be paid to its requirements to improve the future educational outcomes (Muhammad, 2020), In order for the student in the secondary stage to be able to face life's challenges and overcome them with skill and ease, he must own his advanced and renewable thinking abilities, and he also must own the ability to give diverse and multiple solutions to the problems he faces. Therefore, we must concentrate on promoting and developing modern thinking mechanisms for students, who are more mature, have the ability to reach deep degrees of thinking, and are closer to the labor market.

Neural branching thinking strategies are considered one of the effective strategies in the learning and teaching processes because of their excellence in increasing the levels of thinking and training the mind to give quick and proportional responses to the situations surrounding the person. It also stimulates thinking to create new ideas that lead people to creativity. It also activates the role of the learner and encourages him to think through all ways to reach different answers by motivating him with questions and allowing him to express his opinion freely (Omran, 2002; Ali, 2016; Ramadan, 2016; Khalifa, 2018).

The neural branching thinking strategies consist of seven dynamic strategies that are represented in hypothetical thinking, reverse thinking, application of different symbolic systems, symmetry, analyzing the point of view, complementation, and network analysis. These strategies contribute to the flexibility of thinking and follow its paths. Through these strategies, the student can practice thinking with pleasure while teaching and learning school mathematics (Cardellichio & Field, 1997).

Imai (2000), Coskun (2005), Harhash (2016) and Al-Khafaji (2018) stated that neural branching thinking is a type of flexible thinking that leads to the ability to generate multiple ideas with different meanings that help to find solutions to potential problems, and this could be done through training on this thinking and also through training on methods of investigation and exploration, especially that the neural branching thinking support the creation of new connections and branches.

Many studies have confirmed the importance of using neural branching thinking strategies in developing thinking in general, and have also emphasized the ability of neural branching thinking strategies to increase the capabilities and efficiency of the human mind and develop its skills. Shehata's study (2013) indicated the effectiveness of the proposed program that are based on neural branching thinking strategies in developing mathematical communication skills among the primary school students. The study of Abdul Majeed (2015) confirmed the impact of using neural branching thinking strategies in teaching the calculus on the skills of self-organized learning and the assessment of mathematical values among the students of the Faculty of Education.

The research's problem

The researchers felt the research's problem after reviewing the previous studies that were conducted on mathematical proficiency. It became clear to the researchers that most mathematics teachers do not concentrate on developing all dimensions of mathematical proficiency, she also finds out that there may be some interest in developing conceptual understanding, and on the other hand, there is weakness in developing procedural fluency. The researchers also noted that there is a limited interest in developing both of strategic competence and productive disposition. The researchers also felt that students are not allowed to deal with mathematical problems that help in developing the strategic competence, which means that their mathematical proficiency decreases and this in turn affects their performance in the classrooms. Therefore, teachers must possess a very good mathematics skill to improve its learning and to train students on teaching practices continuously to develop dimensions of their mathematical proficiencies (Figgins, 2010; Freund, 2011; Er, 2012; Siegfried, 2012; Hoffmann et al, 2014).

Since the researchers are a teacher for secondary school students, she noticed, by virtue of her work, the weakness of the basic skills in mathematics among the students, and the inability of some of them to make optimal use of mathematical concepts in solving problems related to these concepts. She also noticed a low level of logical thinking in providing explanations and justifications for solutions that are related to the exercises and activities. The researchers noticed a deficiency in the ability of interpreting the verbal problems and represent them correctly. After conducting a test for the mathematical proficiency in the exploratory experiment that was applied to a sample of first-year secondary school students in the third secondary school in Al-Khobar Governorate, the researchers found out the low scores of students in the test, especially in the dimension of strategic competence and the dimension of adaptive reasoning. The average grades of students in the dimension of strategic competence were one degree out of five, and the average grades of students in the dimension of adaptive reasoning were two degrees out of eight. The average degree of the students in the total sum of the test of Mathematical proficiency was 11 out of 25, and this is considered evidence of the weakness of the dimensions of mathematical proficiency among the students in the exploratory sample of the first-year secondary school.

1- To find out the extent of activating neural branching thinking strategies in teaching mathematics at the secondary level, interviews were conducted with a group of mathematics supervisors from the Office of Educational Supervision in Al-Khobar Governorate, and a set of open questions were asked to them, which are as follows:

a- Do the teachers have an idea about the concept of neural branching thinking?

b- Do the teachers apply neural branching thinking strategies?

c- If the teacher applies the neural branching thinking strategies, does she apply them correctly?

d- Does the teacher prefer to formulate test questions according to neural branching thinking strategies?

e- Is the teacher interested in activating the questions of the higher-order thinking skills that stimulate the students' questions?

f- Did activating the neural branching thinking strategies help in improving the teacher's performance in teaching?

After conducting these interviews, the researchers concluded that most mathematics teachers do not have idea about the concept of neural branching thinking, and that few of them are keen to activate neural branching thinking strategies while formulating test questions, but this is not achieved correctly. The researchers also found that some mathematic teachers are not keen to activate the questions of higher-order thinking skills for students because it requires more time and effort from the teacher, and from the students to get appropriate solution, because students are not accustomed to this type of thinking.

Based on the foregoing, and after reviewing previous studies and the results of the exploratory experiments that was carried out by the researchers, the problem lies in the presence of weakness in the dimensions of mathematical proficiency in all the stages, especially in the secondary stage. Moreover, there is a problem lies in activating the neural branching thinking strategies in teaching mathematics correctly among the secondary school teachers. Therefore, work began on formulating mathematics courses, And these courses must contain activities and issues that develop the thinking process, through neural branching thinking strategies, which rely on urging the student to give multiple solutions to one issue, and help her to benefit from the known information in order to create new and innovative ideas, And then work on improving mathematical proficiency, if these strategies are used in the right way, and therefore it is possible to formulate the research problem through the following question:

What is the effectiveness of neural branching thinking strategies in teaching mathematics on developing the mathematical proficiency among students in the first grade of secondary school?

Due to the above question, we can ask the following questions:

1- What is the effectiveness of neural branching thinking strategies in developing the four dimensions of mathematical proficiency: (conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning), and mathematical proficiency as a whole, in the mathematics course of students in the first grade of secondary school?

2- What is the effectiveness of neural branching thinking strategies in developing the productive disposition among students in the first grade of secondary school?

Hypotheses of the research

In order to answer the questions of the research, the following hypotheses were tested:

1- there was significant statistical difference at the indicator level of ($\alpha \leq 0.05$) between the average of experimental group’s students (who studied by the neural branching thinking strategies), and the average of control group’s students (who studied by the teacher's guide way), in the post application for the mathematics skill as a whole, and at every dimension of its dimensions (conceptual understanding, procedural fluency, strategic competence and adaptive reasoning) toward the experimental group.

2- there was significant statistical difference at the indicator level of ($\alpha \leq 0.05$) between the averages of experimental group’s students (who studied with neural branching thinking strategies), and the averages of controlled group’s students (who studied with teacher's guide way) in the post application for the scale of productive disposition toward the experimental group.

Theoretical framework and previous studies

Neural branching thinking strategies

Neural branching thinking strategies are defined as a group consisting of seven dynamic strategies, sometimes cognitive and sometimes metacognitive. It contributes to the flexibility of thought and the multiplicity of its paths and control, and its intentional modification, through which the learner practices the habit of thinking cheerfully during the teaching and learning of school mathematics. Its continuous practice helps in achieving the specified educational goals successfully, and it also helps the student to adapt to different life situations, which are represented in: hypothetical thinking, reverse thinking, using the different symbolic systems, symmetry, point-of-view analysis, complementarity, and network analysis, as shown in Figure (1), (Adam, 2008; Ali, 2009; Al-Harbi, 2015; Ahmed, 2016; Cardellichio & Field, 1997).

Figure 1: The Neural Branching Thinking Strategie

The following is a presentation of these strategies as mentioned by Cardellichio & Field (1997), Omran (2002), Adam (2008), and Muhammad (2018):

1- Hypothetical Thinking Strategy: It is a powerful technique for discovering new information, and stimulating the growth of thinking in different directions, because it forces us to visualize unexpected results and consequences. The main point of using hypothetical questions does not lie in the methods of the question itself, but in the sequence of questions directed by the teacher, which clarify the complexities of things and create events and the network of interrelated relationships that follow (Cardellichio & Field, 1997).

2- Reversal Thinking Strategy: In reverse thinking, it is possible for the student to ask questions about a subject instead of finding the answers, and this helps him to test and examine the presented problem, and from which he can discover new ideas to deal with it from several aspects (Al-Hanan, 2016; Ali, 2009)

3- Application of Different Symbol Systems Strategy: This strategy is used through applying the symbol system to situations that are not usually used, and it is applied to facilitate communication between people to solve a problem, This strategy can be used in the educational situation so that the student will be able to transform a verbal problem into a mathematical equation to facilitate dealing with and solving it, or graphic representation or drawing to clarify the relationships between the data of the problem (Cardellichio & Field, 1997).

4- Analogy Strategy: This strategy is one of the strategies that encourage the student to study the information he possesses and try to find new connections between them in terms of the similarity or difference of its parts, this strategy creates the creativity of the student, because his responses require new insights between the elements (Omran, 2002; Shehata, 2013)

5 - Analysis of Point of View Strategy: the analysis of point of view strategy is a process by which it is determined why a person has a particular opinion or belief, it can be taught to students in a way that stimulate them to ask about the details and evidences, and to express their point of view about the reasons why they hold that opinion, and it is a way to expand the scope of thinking (Omran, 2002).

6- Completion Strategy: This strategy depends on encouraging the student to complete the event or story, which can expand his thinking and raise the question about what is missing in order to get a complete answer or justification for the subject (Cardellichio & Field, 1997; Abdul Azim, 2009).

7- Web Analysis Strategy: The web analysis strategy depends on developing the student's abilities to express these relationships, link them to each other, identify and present them in a simplified way (Cardellichio & Field, 1997; Omran, 2002).

Mathematical proficiency:

Schoenfeld (2007) stated that mathematical proficiency lies in the student's skill of the four aspects of knowledge (conceptual understanding, procedural fluency, strategic competence and adaptive reasoning) in testing the strategies of appropriate solution, which show what can be done and what is inclined to be done mathematically while solving mathematical problems.

Groth (2017) also indicated that teachers should help students in learning mathematics efficiently, and that the concept of mathematical proficiency arose from a trend whose goal was returning back to understand the basics of mathematics and to think about mastering the skills by focusing on the student's ability to memorize, remember, and perform algorithms.

The National Research Council [NRC] has identified five essential elements that make up the Strands of Mathematical Proficiency and stated that these dimensions help students to engage in successfully understanding mathematics, which are conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition, and the following figure (2) illustrates those dimensions:

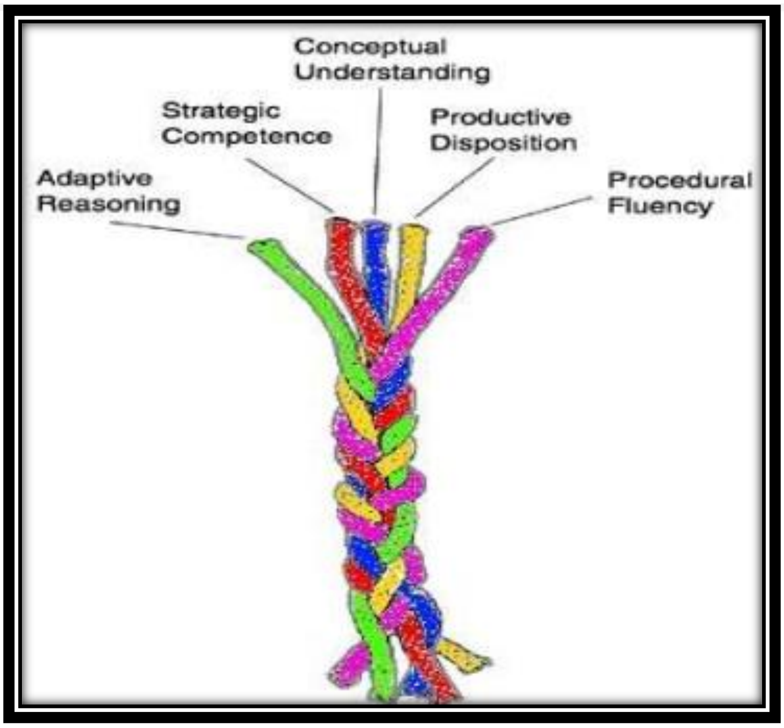

Figure 2: The Strands of mathematical Proficiency (NRC, 2001)

These five dimensions must work together to provide the students with the concept of mathematical proficiency. In the traditional way in which students learn, procedural fluency is the dominant element in defining mathematical proficiency, but when the concept of mathematical proficiency expands, we find that the first four dimensions of mathematical proficiency need to be developed continually to ensure that students can master mathematics (Suh, 2007). Mathematical proficiency is a multidimensional concept and cannot be achieved by paying attention to one or two of these dimensions, actually the five dimensions together provide a framework for the knowledge, skills, abilities, and beliefs that make up mathematical proficiency (Samuelsson, 2010; Bergem & Pepin, 2013).

I will present now the dimensions of mathematical proficiency in detail:

1- Conceptual Understanding: It is the comprehension of mathematical concepts and the relationships and operations between them, it means that the student forms an integrated understanding of mathematical ideas, so he understands the facts, relationships and mathematical methods that he learns, in addition to understanding the importance of mathematics to learn new ideas, and solve the problems he faces (NRC, 2001).

2- Procedural Fluency: Procedural fluency refers to knowledge of procedures, how to use them, and when to use them appropriately. Procedural fluency is presented in the skill of remembering and performing procedures with flexibility, accuracy and efficiency, and linking them together to produce the largest possible number of solutions and making sure of their effectiveness. In addition, dealing with many tasks that involve mathematics in daily life requires fluency in dealing with algorithms to perform arithmetic operations mentally or in writing (NAEP, 1997; Obeida, 2017; Al-Hanan, 2018; Al-Harbi, 2019; Al-Shammari, 2019; Al-Juhani, 2020; Al-Badri, 2021).

3- Strategic Competence: Strategic competence is the ability to reformulate the mathematical problem, represent it, solve it, and form mental images of it. It includes solving non-routine problems that require higher thinking skills, so students must have flexibility in dealing with various types of problems, not only routine ones, as flexibility develops through expanding the limit of knowledge to solve the non-routine problem as well (NRC, 2001; MacGregor, 2013; Al-Dhani, 2017; Yulian & Wahyudin, 2018; Al-Saeed, 2018; Ben Merdah, 2019; Abdel-Fattah, 2020; Muhammad, 2020).

4- Adaptive Reasoning; Adaptive reasoning refers to the ability to think logically about the relationships between concepts and situations, provided that this reasoning is valid, acceptable and is derived from careful thinking about the alternatives and from knowledge of how to justify the results. In mathematics, adaptive reasoning is the basis that gathers everything, and is the shining star that guides the learning process, and one of its uses is to move between many facts, concepts, strategies and solutions to see if they can fit together in a logical way. In addition, adaptive reasoning is a form of inductive thinking (mathematical inquiry), and the student can master inference as soon as he possesses sufficient knowledge and when the mathematical problem is clear and understandable (NRC, 2001).

5- Productive Disposition: Productive disposition refers to understand the meaning of mathematics and feeling that it is useful worthwhile, feeling its beauty, appreciating its work inferring its importance through steps of scientific and theoretical reasoning, and believing that the efforts made in learning mathematics will pay off. Then the student feels that he is active and interactive. If students want to develop their abilities in conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning, they must believe that mathematics is understandable, not complex, meaning that with constant study there will be the results of trying to understand mathematics, and this helps in building their mathematical proficiency, and then It contributes to create a strong community of mathematics students (NRC, 2001; NRC , 2005; Figgins, 2010; Al Harbi, 2019; Al-Maliki & Al-Riyashi, 2019).

Based on the above, the researchers understand the importance of integrating the five dimensions of mathematical proficiency, so that they should be developed and enhanced simultaneously with each other, Because focusing on one dimension without the other dimensions does not help in understanding mathematics adequately, and therefore the student will face an obstacle in applying it to life problems, the following is a summary of the concept of mathematical proficiency as a whole, as shown in figure (3) below:

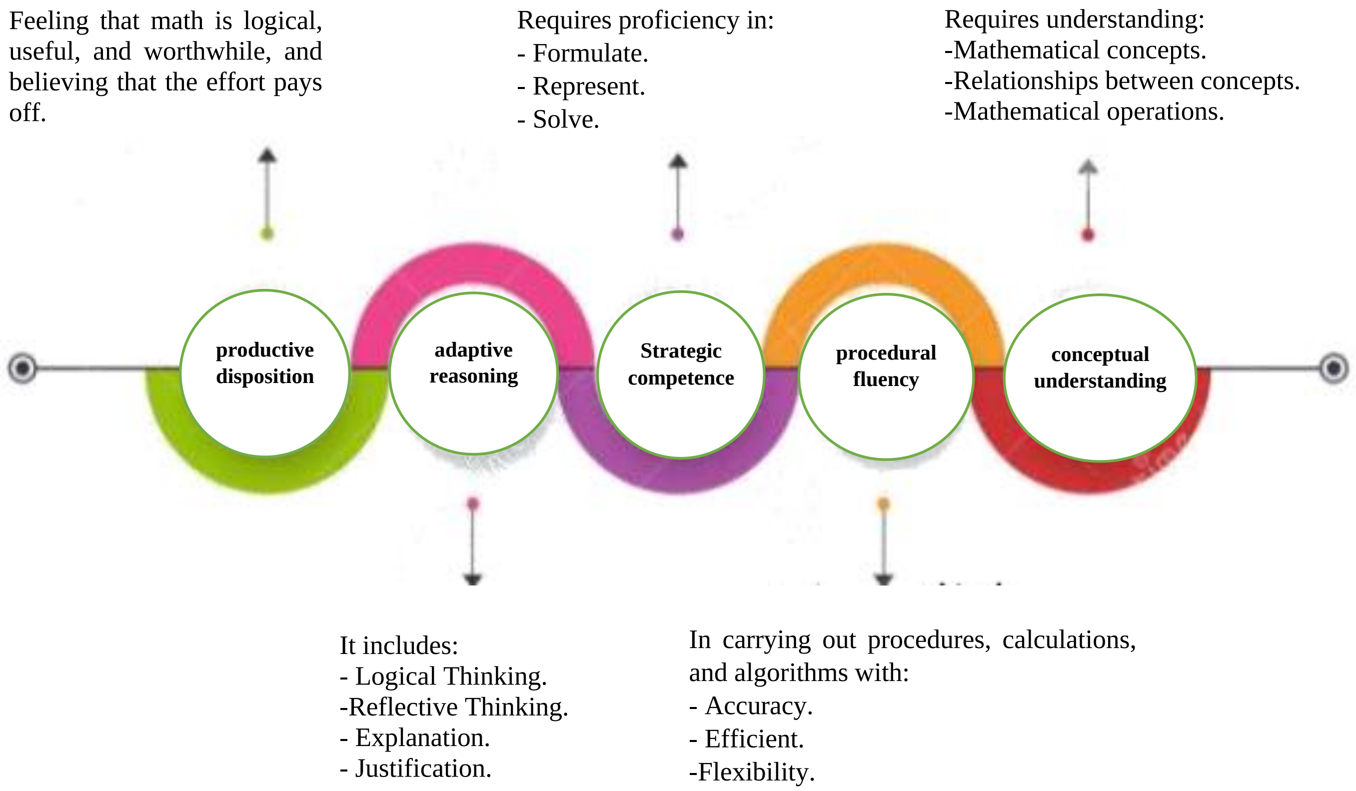

Figure (3) summary of the concept of dimensions of mathematical proficiency

Several studies have confirmed the importance of developing programs for mathematics to develop dimensions of mathematical proficiency, such as the study of Flynn et al. (2002), which demonstrated the effectiveness of using computers in solving algebraic problems in developing mathematical proficiency among secondary school students, more than solving by manual methods. The study of Schoenfeld (2007) revealed the effectiveness of a training program for developing students' mathematical proficiency and how to evaluate its dimensions. Also, the study of Kim & Chang (2010) demonstrated that the students of English language course who used computers to learn mathematics outperformed their English-speaking peers in mathematical proficiency.

Methodology:

This research aims at revealing the effectiveness of using neural branching thinking strategies in teaching mathematics to develop mathematical proficiency among secondary school students. To achieve this aim, the research adopted the experimental method with a quasi-experimental design, which is based on designing two groups, an experimental group and a control group; figure (4) shows the experimental design of the research.

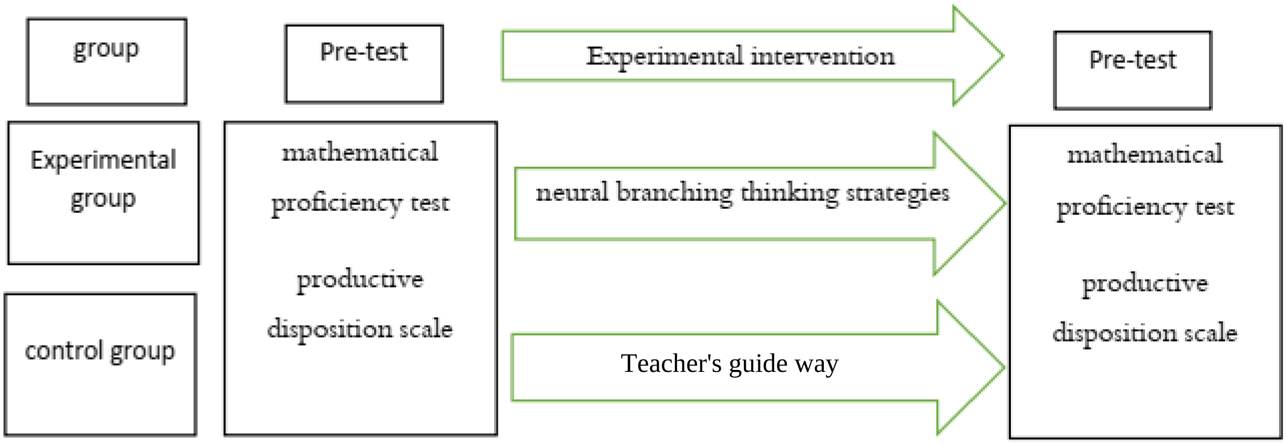

Figure (4): The quasi-experimental design of the research

Population research and its sample:

● **population research:** The population research included all students in the first secondary grade (first level) from secondary schools in Al Khobar Governorate in the Eastern Province in the first semester of the year 1442 AH, they were 2888 students, according to the statistics of the Statistics Department of the Education Statistics and Decision Support Center at the Ministry of Education.

● **Research sample:** a sample of first-year secondary school students from the eighth secondary school in Al-Khobar was selected, and two divisions were selected out of five divisions that were in the school of the first secondary grade, one of them representing the experimental group, and the other the control group. This selection was done through random drawing. The first group was chosen to represent the experimental group and the number of its students is 31, and it studies the units of "parallel and Perpendicular" and "Congruent Triangles" using neural branching thinking strategies, and the fifth division that represents the control group, the number of its students is 32, and it studies the units of "parallel and Perpendicular" and "Congruent Triangles" by the method of the teacher's guide, and thus the whole sample contains (63) students.

Tools of the research:

Test of mathematical proficiency:

By reviewing the literature and previous studies that focused on developing dimensions of mathematical proficiency, it was found that measuring the first four dimensions of mathematical proficiency: (conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning) is done by preparing a test that measures the cognitive and skill aspect of students. The researchers have benefited from some of these studies, in preparing the Mathematical Proficiency Test, such as: Dhlamini & Luneta (2016), Al Dhani (2017), and Obeida (2017).

Based on the analysis of the scientific content of the two units of "parallel and Perpendicular" and "Congruent Triangles" according to the dimensions of mathematical proficiency, the cognitive aspects were identified in the two units that will measure the four dimensions of mathematical proficiency, and then the questions were formulated in a way that ensures the measurement of each of these dimensions, The test was divided into four parts so that the questions of each dimension is separated from each other, and the total number of the questions was 34. the initial image of the Mathematical Proficiency Test was presented to a group of arbitrators specialized in courses and methods of teaching mathematics in order to explore their views about the appropriateness of the paragraphs of the mathematical proficiency test for the students in the first grade of secondary school, All the necessary modifications were done according to the arbitrators' instructions.

Exploratory test: After confirming the validation of the arbitrators, the test was applied to an exploratory sample of students in the first grade of secondary school in the third secondary school in Al-Khobar Governorate; they were 36 students in the first semester of the year 2019/2020. in order to determine the required average time, the time was defined according to the time of first student who finished her test and the time of last students in the same test and calculating the average of the two times. Thus, the time required to perform the test is (85) minutes. And to verify the internal consistency of the Mathematical Proficiency Test by calculating the Pearson correlation coefficient, to find out how strongly each of the test questions is related to the total degree of the skill to which it belongs, and the results came as shown in the two tables (1) and (2):

Table 1

Correlation coefficients between the question’s degree in the Mathematical Proficiency Test and the total degree of the dimension that measures (the exploratory sample contained 36 students)

CONCEPTUAL UNDERSTANDING		PROCEDURAL FLUENCY		STRATEGIC COMPETENCE		ADAPTIVE REASONING	
TEST QUESTIONS	correlation coefficient	test questions	correlation coefficient	test questions	correlation coefficient	test questions	correlation coefficient
1	0.904**	11	0.768**	19	0.840**	25	0.629**
2	0.904**	12	0.640*	20	0.910**	26	0.843**
3	0.839**	13	0.744**	21	0.897**	27	0.860**
4	0.664**	14	0.846*	22	0.745**	28	0.738**
5	0.904**	15	0.819**	23	0.794**	29	0.917**
6	0.904**	16	0.861**	24	0.347*		

7	0.763**	17	0.706**
8	0.348*	18	0.793**
9	0.839**		
10	0.686**		

Note: (* the value of the correlation coefficient is a function at the 0.05 level), (** the value of the correlation is a function at the 0.01 level).

Table 2

Correlation coefficients between the degree of dimension and the total degree of the Mathematical Proficiency Test (Exploratory sample of 36 students)

THE DIMENSION	**CONCEPTUAL UNDERSTANDING**	**PROCEDURAL FLUENCY**	**STRATEGIC COMPETENCE**	**ADAPTIVE REASONING**
CORRELATION COEFFICIENT	0.589**	0.853**	0.845**	0.663**

Note: (* the value of the correlation coefficient is a function at the 0.01 level)

It is clear from Tables 1 and 2 that the values of the correlation coefficients between the question degree and the total degree of the dimension, or between the degree of dimension and the total degree of the test are all significant at the indicator level of (0.01) and (0.05), and this confirms the formative validity of the test.

The stability of the test was also calculated by calculating the stability of each of the four dimensions separately and then calculating the total stability of the test by using the alpha-Cronbach coefficient, the value of the stability coefficient for the four dimensions and for the test as a whole was as shown in the following table 3:

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Table 3

Reliability coefficients for each of the four dimensions of the mathematical proficiency's test as a whole by Cronbach's Alpha method (exploratory sample contained 36 students)

THE DIMENSION	CONCEPTUAL UNDERSTANDING	PROCEDURAL FLUENCY	STRATEGIC COMPETENCE	ADAPTIVE REASONING	THE TEST AS A WHOLE
STABILITY COEFFICIENT	**0.904**	**0.781**	**0.861**	**0.816**	**0.910**

It is obvious from Table 3 that the value of the reliability coefficient of (0.910) is a value confirms that the Mathematical Proficiency Test has an acceptable degree of reliability, and can be applied for scientific research purposes.

The scale of productive disposition:

The literature and previous studies that dealt with the productive disposition scale tool towards mathematics were reviewed and used in order to determine the axes of the scale and the statements of each axis. These studies include Awofala (2017), Al-Omari (2019), and Hutajulu et al. (2019), and the following axes were adopted: tendency to learn mathematics, tendency to mathematics, tendency to enjoy mathematics, tendency to the usefulness of mathematics and tendency to the teacher of mathematics. five-point Likert Scale was used, where each statement includes five tests (strongly disagree, disagree, neutral, agree, and strongly agree), and the scale was presented to a group of arbitrators specialized in courses and mathematics teaching methods, and the necessary measures were taken according to the instructions of the arbitrators' opinions

After confirming the validity of the scale, it was applied to an exploratory sample that contained 33 students of the first secondary grade in the third secondary school in Khobar from the first semester of 2019/2020, in order to calculate the structural validity of the test by calculating the internal consistency coefficient between the degree of each axis and the total degree of the scale, the results were as in Table 4:

Table 4

Correlation coefficients between the degree of axis and the total degree of the scale of productive disposition (the exploratory sample contained 33 students)

THE AXIS	TENDENCY TO LEARN MATHEMATICS	TENDENCY TO ENJOY MATHEMATICS	TENDENCY TO THE NATURE OF MATHEMATICS	TENDENCY TO THE USEFULNESS OF MATHEMATICS	TENDENCY TO THE TEACHER OF MATHEMATICS
CORRELATION COEFFICIENT	0.776**	0.811**	0.802**	0.808**	0.759**

It is obvious from Table 4 that the values of the correlation coefficients between the degree of the axis and the total degree of the scale are all significant at the indicator level of (0.01), This achieves the formative validity of the scale. In addition, the stability of the productive disposition scale was confirmed, where Cronbach's alpha coefficient was used to calculate the stability of each of the five axes separately, and then calculate the overall stability of the scale. as you can see in the table 5.

Table (5)

Stability coefficients for each of the five axes and for the scale of productive disposition as a whole (the exploratory sample contained 33 students).

THE AXIS	TENDENCY TO LEARN MATHEMATICS	TENDENCY TO ENJOY MATHEMATICS	TENDENCY TO THE NATURE OF MATHEMATICS	TENDENCY TO THE USEFULNESS OF MATHEMATICS	TENDENCY TO THE TEACHER OF MATHEMATICS	THE SCALE AS A WHOLE
STABILITY COEFFICIENT	0.793	0.859	0.896	0.909	0.887	0.956

It is obvious from Table 5 that the overall reliability coefficient of the scale was 0.956, which is an acceptable value. Accordingly, it is obvious that the scale of productive disposition and its results are reliable.

Procedures of teaching by neural branching thinking strategies

The teacher's manual was prepared for teaching the units "parallel and Perpendicular" and "Congruent Triangles" to students in the first grade of secondary school after reformulating both units using neural branching thinking strategies. The teacher's manual contains the following contents: (The manual's introduction, the manual's objectives, an overview of neural branching thinking strategies, general directions for the teacher, the proposed teaching time plan and lesson planning) and all the plans were prepared according to the following stages:

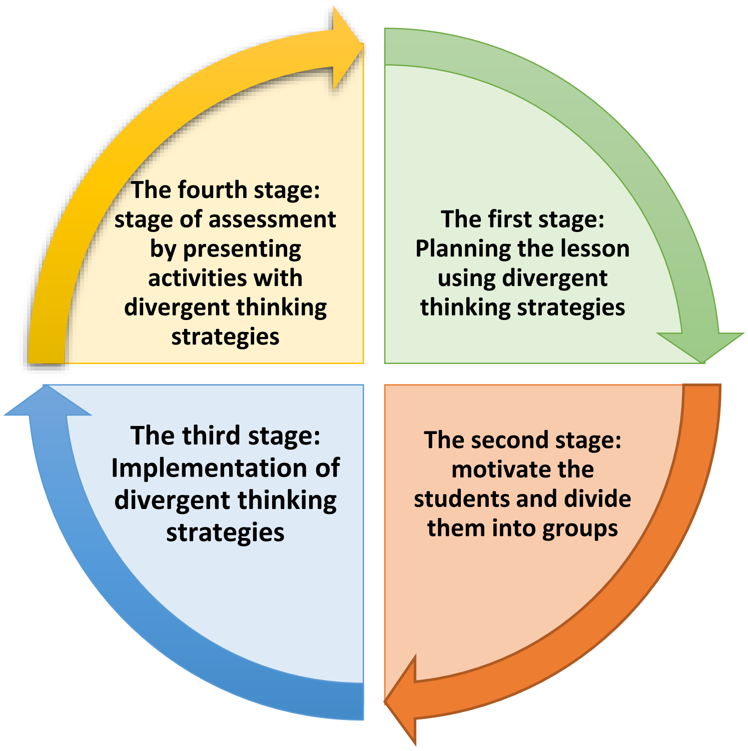

Figure (5) The Neural Branching Thinking Strategies in Teaching Mathematic

After completing the preparation of the teacher is manual, it was presented to a group of arbitrators to obtain their opinions from several aspects, by preparing a form in which the teacher's manual is judged through several specific elements, and then the amendments were made according to the arbitrators' opinions.

Research results and its discussion:

In the following a presentation for the research results that were identified to answer the questions of the research and making sure of its hypotheses:

1- The results that are related to the answer of the first research question:

in order to answer the first research question, what is the effectiveness of neural branching thinking strategies in developing the four dimensions of mathematical skill (conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning), and mathematical skill as a whole, in the mathematics course of students in the first grade of secondary school? The first hypothesis of the research was verified, which states that: there was significant statistical difference at the indicator level of ($\alpha \leq 0.05$) between the average of experimental group's students (who studied by the neural branching thinking strategies), and the average of control group's students (who studied by the teacher's guide way), in the post application for the mathematical proficiency as a whole, and at every dimension of its dimensions (conceptual understanding, procedural fluency, strategic competence and adaptive reasoning) toward the experimental group.

In order to test the validity of this hypothesis, the mathematical proficiency test was applied to the experimental and control groups after implementing the experiment, The value of the t-test was calculated for two independent samples, to identify the significance of the difference between the mean degrees of the students of the experimental and control groups in the post application of the test of mathematical proficiency as a whole and at each of its dimensions (conceptual understanding, procedural fluency, strategic competence and adaptive reasoning). To measure the size of impact of the experimental treatment, the eta square (η^2) and its corresponding d-value were calculated, to identify the size of the impact of using neural branching thinking strategies in developing mathematical proficiency as a whole and at each of its dimensions (conceptual understanding, procedural fluency, strategic competence and adaptive reasoning) and the following table 6 illustrates these results:

Table 6

t-value to indicate the difference between the mean degrees of the students of the control and experimental groups in the post-application of the test of mathematical proficiency as a whole and at each of its dimensions (conceptual understanding, procedural fluency, strategic competence and adaptive reasoning) as well as the Eta square (η^2) and the corresponding d effect size.

THE DIMENSION	THE GROUP	THE NUMBER	ARITHMETIC MEAN	STANDARD DEVIATION	CALCULATED (T) VALUE	DEGREES OF FREEDOM	α SIG.	ETA SQUARE (H2)	D EFFECT SIZE
CONCEPTUAL	Experimental	31	6.968	2.273	4.301	61	0.000	0.233	1.102
UNDERSTANDING	control	32	4.594	2.108					Big
PROCEDURAL	Experimental	31	6.847	2.760	4.322	61	0.000	0.235	1.108
FLUENCY	control	32	3.352	3.577					Big
STRATEGIC	Experimental	31	5.516	2.039	4.431	61	0.000	0.243	1.133
COMPETENCE	control	32	3.188	2.129					Big
ADAPTIVE	Experimental	31	3.895	1.756	5.628	61	0.000	0.342	1.441
REASONING	control	32	1.539	1565					Big
THE TEST AS A	Experimental	31	23.226	7.719	5.207	61	0.000	0.308	1.334
WHOLE	control	32	12.672	8.344					Big

It is obvious from table 6 the following results:

- The values of the t-test were statistically significant at the indicator level of ($\alpha \leq 0.01$), which indicates the existence of a significant statistical difference between the mean scores of the students of the experimental and control groups in the post application of the test of mathematical proficiency as a whole and at each of its dimensions (conceptual understanding, procedural fluency, strategic competence and adaptive reasoning) toward the experimental group students, and accordingly, the alternative hypothesis is accepted and the null hypothesis is rejected.

- The size of impact of neural branching thinking strategies on developing mathematical proficiency as a whole and at each of its dimensions (conceptual understanding, procedural fluency, strategic efficiency and adaptive reasoning) is large, and this indicates the effectiveness of neural branching thinking strategies in developing mathematical proficiency as a whole and at each of its dimensions (conceptual understanding, procedural fluency, Strategic competence and adaptive reasoning) among students in the first grade of secondary school in mathematics course.

The reason of this result is that the neural branching thinking strategies provide more opportunities for understanding and comprehension that helps students to employ their mathematical learning experiences in various mathematics topics, And because it also facilitates the opportunities of stimulating the brain, the multiplicity of visions and branching the pathways of thinking, and gives the ability to control and modify these pathways, which raises the level of efficiency of the human brain and increases its capabilities, It also increases students' awareness of what they are studying in a particular situation and how they learn and to what extent their learning has reached, as well as helping to focus on cognitive processes. The result of this hypothesis is similar to the findings of some studies that showed that students who studied using neural branching thinking strategies had better results, comparing with the students who studied using the teacher's guide way, such as Shehata Studies (2013), Al-Mansoori (2017), Al-Badri (2019), Al-Baqami (2019), Jebur (2020), Salha (2020), and Muhsin & Al-Ibad (2020).

2- The results that are related to the answer of the second research question:

in order to answer the second research question: what is the effectiveness of neural branching thinking strategies in development of productive disposition among students in the first grade of secondary school? The second hypothesis of the research was verified, which states that: There is a statistically significant difference at the indicator level of ($\alpha \leq 0.05$) between the average degrees of the students of experimental group (which studied with neural branching thinking strategies) and the average degrees of the students of control group (which studied by the teacher's guide way). In the post application of the productive disposition toward the experimental group.

in order to test this hypothesis, the productive disposition scale was applied to the experimental and control groups, after implementing the experiment, and the t-test values were calculated for two independent samples, to identify the significance of the difference between the mean degrees of the students of the experimental and control groups in the post application of the productive disposition scale. Eta square and its corresponding d values were calculated to know the size of the impact of using neural branching thinking in developing productive disposition. Table 7 shows these results:

Table 7

T-value to indicate the difference between the mean degrees of the students of the control and experimental groups in the post-application of the criterion productive disposition, Eta square and the corresponding d effect size.

THE GROUP	THE NUMBER	ARITHMETIC MEAN	STANDARD DEVIATION	CALCULATED (T) VALUE	DEGREES OF FREEDOM	α SIG.	ETA SQUARE (H2)	D EFFECT SIZE
EXPERIMENTAL	**31**	**138.10**	**21.341**	**2.236**	**61**	**0.029**	**0.076**	**0.574**
CONTROL	**32**	**125.84**	**22.127**					**Average**

It is obvious from Table 7 that the values of the t-test are statistically significant at the indicator level of ($\alpha \leq 0.05$) in the scale of productive disposition, which indicates that there is a statistical significant difference between the mean degrees of the students of the experimental and control groups in the post-application of the scale of productive disposition as a whole toward the students of the experimental group, which leads to acceptance of the second hypothesis of the research. The results also indicate that the size of the impact was of medium degree, because the values of eta square (η^2) are greater than 0.06, where the values of d = 0.574 and the value of eta square (η^2) is 0.076, and this means that the total variance of the dependent variable (productive disposition) is due to the influence of the independent variable (neural branching thinking strategies).

The research findings of the experimental group's superiority over the control group in the dimensional application of the scale of productive disposition as a whole is considered an evidence of the importance of developing dimensions of mathematical proficiency during the teaching of mathematics, including the development of productive disposition, as mentioned by kepner & Huinker (2012), one of the benefits of developing the dimensions of mathematical proficiency is that when the student realizes his ability to link mathematical concepts and integrate them with his previous learning, he automatically tends to develop himself in mathematics, and increases his confidence in his ability to understand, interpret and express his views, which improves students' inclinations towards mathematics and mathematical concepts.

MacGregor (2013) and Al-Hanan (2018) stated that the importance of developing the dimensions of mathematical proficiency lies in giving the students confidence in themselves and that it generates a desire to face challenges and difficult situations, which increases students' inclinations towards mathematics, and thus develops their ambition in the future to research in order to develop their higher skills in mathematical proficiency.

according to the foregoing, it can be said that the using the neural branching thinking strategies provided an opportunity to encourage the students to develop their personalities and the tendency to express opinions and respect the opinion of others. It also encouraged them to take the initiative to ask questions and discuss with each other.

Recommendations and suggestions

According to the previous discussions, and the results of the effectiveness of neural branching thinking strategies in teaching mathematics on developing dimensions of mathematical proficiency for secondary school students, we can get several recommendations such as: using the neural branching thinking strategies by mathematics teachers in teaching mathematics at different grades, for their effective role in providing the opportunity for students to reach higher levels of thinking, and their ability to give a deep understanding of mathematical principles and laws, which develops their inclinations towards a mathematics course, and thus achieves an actual growth in the dimensions of mathematical proficiency they have. Guiding mathematics teachers to work on developing the dimensions of mathematical proficiency when teaching mathematics, through using different learning strategies that support this, such as neural branching thinking strategies. Guiding mathematics teachers to interest in developing the emotional side while teaching mathematics and try to enhance it among students through the application of different teaching strategies, including neural branching thinking strategies. Guiding mathematics teachers to create the educational environment and provide the necessary capabilities to activate neural branching thinking strategies in teaching mathematics to be stimulating to the development of mathematical proficiency.

The current study suggests conducting another and similar study that concentrates on studying the challenges that obstacle using neural branching thinking strategies in teaching mathematics at the secondary grade according to the viewpoint of mathematics teachers at the secondary stage. It suggests conducting more studies to verify the reality of teaching practices that support neural branching thinking strategies in teaching mathematics among mathematics teachers at the secondary stage. It also suggests conducting more studies to verify the effectiveness of using neural branching thinking strategies in teaching mathematics on developing mathematical proficiency among students of special groups, such as those with learning difficulties, the gifted, the blind, the deaf and dumb. and also suggests conducting more studies to reveal the effectiveness of using neural branching thinking strategies in developing mathematical proficiency, and other learning outcomes that are related to a specific aspect of thinking, and also have a link to dimensions of mathematical proficiency, such as: solving mathematical problems, critical thinking, creative thinking, geometrical thinking and logical thinking.

Conclusion

The aim of the current research is to identify the effectiveness of neural branching thinking strategies in teaching mathematics on developing mathematical proficiency among secondary school students. The problem of the research was the weakness in the level of mathematical proficiency among secondary school students. To solve this problem, the following question was asked: What is the effectiveness of neural branching thinking strategies in teaching mathematics on developing the mathematical skill among students in the first grade of secondary school? The experimental method with a quasi-experimental design was used, and the research sample consisted of (63) students of the first secondary grade in Al-Khobar Governorate, Saudi Arabia. The sample was divided into two groups, one of them: an experimental group (studied using neural branching thinking strategies) contained 31 students, and a control group (studied using the teacher's guide way) contained 32 students. The experimental materials that contained teacher's manual were designed through using neural branching thinking strategies. The research tools were prepared to collect data, they are as follows: the Mathematical Proficiency Test and the Productive Disposition Scale. After collecting the data and using statistical methods to process it, the results showed that the students of the experimental group outperformed the students of the control group in both of mathematical proficiency and tendency to mathematics. This confirms what was mentioned in the study of Tsigankov & Koulakov (2009) that the neural branching thinking strategies motivate to think in different and diverse directions, through unfamiliar questions, which push the student to search for multiple solutions and answers to the same question, which achieves the best possible results. Abdul Majeed (2015) stated in his study that the neural branching thinking strategies contributed to improving the level of planning skill for the aim, and the ability to memorize and remember among secondary school students. Ahmed's study (2016) indicated that the neural branching thinking strategies increase the positivity of secondary school students in learning mathematics, and this in turn led to the superiority of the experimental group students in both of mathematical proficiency and productive disposition. This result might be interpreted by the training provided by the learning environment with neural branching thinking strategies for students to think in different directions, which makes learning better than the traditional method. The researchers suggest conducting researches similar to this research on other grades, to identify the effectiveness of neural branching thinking strategies on students' attitudes in various courses.

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