The Elaboration Study as an Innovative Learning Model in an Effort to Improve the Understanding of Mathematics

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Abstract

The elaboration of a learning model between a Team Assisted Individualisation learning model and Co-Op Co-Op learning model aims to improve the students’ understanding of mathematics, which consists of conceptual and procedural understanding with an emphasis on the importance of 4C (critical thinking, creative thinking, collaboration, and communication) skills, and a proficiency in accordance with Education 4.0. A true experimental design was conducted in this study with random cluster sampling techniques for grade X senior high school science students. The analysis of t-independent test resulted that $\alpha > p$ (0.05 > 0.001). It indicated a difference of the students' understanding of mathematics between the experimental class and the control class after the application of the learning model. The average increase in the students' understanding of mathematics on the experimental class was 0.73 (high category), while the average increase in the control class is 0.55 (medium category). These results were confirmed by the preferences of the students’ positive response of 3.23, after the learning process. The elaboration was improving the
students' understanding of mathematics through knowledge design, with critical and creative
thinking and communication processes through the exchange of ideas in the problem-solving
phase so that students have a better and more thorough understanding. Moreover, the
elaboration model could be an alternative to a learning model to increase students' understanding
of mathematics.

**Keywords:** Elaboration of learning models, Education 4.0, Conceptual and procedural understanding.

**Introduction**

Rapid and dynamic development along with technological advances has led to changes in
the learning paradigm with the importance of mastering 4C skills (critical thinking, creative
thinking, collaboration, and communication) in the Education era 4.0. The demand for mastery
of 4C skills lately requires teachers to design a learning model which is an innovative, meaningful
and enjoyable one. Directed learning encourages students to find out through the use of
information from various sources that also leads to exercise analytical thinking in decision making
that emphasises cooperation and collaboration. It is intended to explore interests, talents, and
potentials to obtain students who can apply their knowledge in everyday life in the era of
Education 4.0.

Mathematics is one of the basic sciences that has an essential role in human life. Mathematics becomes a compulsory subject which aims to prepare students to be able to deal with changing conditions and respond to them; in accordance with national education goals. However, many students still feel inferior to mathematics which makes them feel depressed in trying to understand and study the subject. Lack of self-confidence, competence, and negative attitudes are closely related to failure to understand lessons (Ali & Reid, 2012) that have an impact on learning achievement (Ismail & Awang, 2012). It can be seen from the accomplishment of TIMSS 2015 with a math score of 397, placing Indonesia in 45th place out of
50 countries, as well as in the 2015 PISA results ranking the mathematics abilities of Indonesian students 63rd out of 72 countries (Mullis, Martin, Foy, & Hooper, 2015; OECD, 2016). Low mathematical skills of students are related to the poor teaching quality that does not emphasise conceptual meaning and understanding, and the conventional learning method which is teacher-centred and boring (Tarigan, Surya, & Yusnadi, 2017; Zakaria, Addenan, Maat, & Nordin, 2016).

Based on the results from the observations in public Senior High Schools (SHS), 2 Sinjai, the students’ mathematical abilities were still low which could be seen from the analysis of students' daily test scores. Only 47% of the students have reached the specified educational standard which was 75. Students have difficulty in solving problems in the form of applying concepts in everyday life. One reason for the difficulty of students is the lack of students' understanding of the concepts. The learning process is still teacher-centred such that a teacher gives the theory and continues with examples of questions to be solved by students. The learning model means students tend to be passive, caused by students depending on the teacher who gives the theory rather than finding out the process of solving and decision making. Whereas in building students' mathematical abilities, it needs mathematical understanding in improving mental construction and experience (Prediger, 2010) so that students do not only understand procedurally but also conceptually the theory to solve a problem (Rittle-johnson & Schneider, 2015). Conceptual and procedural understanding is essential (Dane, Çetin, Baş, & Sağırlı, 2016) and both are the predictors of mathematical achievement (Gilmore, Keeble, Richardson, & Cragg, 2017); moreover, through mastery of conceptual and procedural understanding, students will have knowledge of concepts, principles and definitions and procedures including sequence of actions and algorithms used in problem-solving (Star & Gabriel, 2013).

Based on those problems, it requires a learning innovation that can involve the student optimally with the ability of the students’ understanding of mathematics and application of the theory. One alternative innovation is by elaborating the model in order to have a proper, optimal,
meaningful and enjoyable learning procedure. The elaboration is conducted by covering the weaknesses of a model implemented or complemented by other learning models with their advantages. In this study, the elaboration in order to accommodate the education objectives 4.0 is the Team Assisted Individualisation (TAI) learning model and Co-op Co-op learning model.

The weaknesses of the TAI learning model (Slavin, 2015; Warsono & Hariyanto, 2017) can be covered with the Co-Op Co-Op learning models, namely (1) TAI learning models only choose one assistant so that students who don't understand solely depend on the friend in their group (assistant). Likewise, the presentation of group work results is still dominated by the assistant or student who has a high understanding, so that students with less knowledge are not motivated to be active. In the Co-Op Co-Op learning model, it does not use an assistant which encourages student motivation and responsibility because of equal liable opportunity. (2) More likely in individuals resulted tolerances in cooperation and collaboration is lacking. In the Co-Op Co-Op learning model, it uses assessments that focus on group achievement.

On the other hand, the weakness of the Co-Op Co-Op learning model (Slavin, 2015; Warsono & Hariyanto, 2017) can be enhanced with TAI learning model, namely (1) the share of a topic is determined by each group, so students only master one subject. In the TAI learning model, the assignment of each group is different but with the same concept. (2) The assessment process is only in groups so that many students feel dissatisfied with the results obtained. It is complemented by a TAI learning model that uses individual evaluation.

The elaboration of the TAI and Co-op Co-op learning models is one of the alternatives for learning processes to accommodate Education 4.0 which requires students to be able to collaborate, communicate, think creatively and critically and be able to solve problems without ignoring student character education (Hussin, 2018). The elaboration study of this model is in accordance with what was stated by Díaz (2017), that in the process of knowledge, transfer is not only conducted with a series of practical and conceptual skills but it must be integrated with
elements of character education such as motivation, ethics and other social attitudes, and attitudes. Therefore, the elaboration of this model is expected to support learning in the education era 4.0 which promotes teaching and learning processes to be more interactive, exciting and fun, and technology-based (Mendezabal & Tindowen, 2018).

Materials and Method

The study was performed in public SHS 2 Sinjai with 52 students as the subjects. The students were divided into 26 students for the experimental class and 26 students for the control class obtained by a cluster random sampling technique on class X science students. The instruments used were a mathematics test consisting of six (6) essay questions to measure the students' understanding and also a questionnaire to analyse students' preferences. Analytical data were conducted in this study using a normality test, homogeneity test, hypothesis test, and normalised gain. The quantitative approach with nonequivalent control of group design was used in this study with the following pattern.

![Figure 1. The research design](image)

Annotation:

O1: Pre-test result of the experiment class

O2: Post-test result of the experiment class
O3: Pre-test result of the control class
O4: Post-test result of the control class
X1: Treatment of the elaboration of TAI and Co-Op Co-Op learning model
X2: Treatment of TAI learning model

The elaboration of the TAI and Co-op Co-op learning model not only prioritises knowledge aspects but also prioritises affective and psychomotor aspects of students who in the process can increase their individual self-esteem. Because the acceptance of greater individual differences, students learn how to collaborate in a group from which can emerge kindness, sensitivity, and tolerance. Giving equal opportunities in learning will eventually develop a positive attitude and confidence in students. A reward is not only for the groups but also provides to individuals, so students are motivated to be more active in understanding teaching materials. Learning through groups and presentations makes learning time more effective. The elaboration stage of the TAI and the Co-Op Co-Op learning model is shown below.
Figure 2. The elaboration stage of the TAI and the Co-Op Co-Op learning model

(Source: Huda, 2014; Slavin, 2015; Warsono & Hariyanto, 2017)

The essential components of the elaboration of this model are (1) Collaboration when students group heterogeneously; (2) Creative and critical thinking when the teacher gives assignments individually and evaluates individually; this is intended for students to be able to think creatively and critically in understanding the problems given; (3) Problem Solving when the tasks that have been done are evaluated and discussed in groups to find solutions to problems; and (4) Communication when the results of the discussion are presented with an appointment or
sweepstake system so that all students are always motivated to be ready to communicate and share the results of their arguments.

The ability of students' understanding of mathematics is evaluated by two aspects, namely conceptual understanding and procedural understanding aspects. Conceptual and procedural understanding is part of the students' skills in learning mathematics (Sahin, Yenmez, & Erbas, 2015) and is very important for competence in mathematics (Habila, 2017). According to Rittle-johnson & Schneider (2015), these two understandings have two-way (bi-directional) relationships and mutual support in which procedural understanding indicates the use of procedures in problem-solving, whereas conceptual understanding is basically a concept knowledge whose level of connection reflects someone with expertise. Mainini & Banes (2017) explained that conceptual understanding is the ability of students to recognise and understand the core ideas and to identify how these ideas are interrelated, while procedural knowledge is the ability of students to carry out the steps needed to solve problems. The relationship between procedural and conceptual understanding can be constructed through the representation of issues and problem solving (Kadijevich, 2018). This understanding ability encourages students in cognitive processes that make it possible to build broader and more complex mathematical understandings. Mastery of this ability will have an impact on the application of knowledge studied at school into daily life applications.

To measure the students' understanding of mathematics, an instrument test related to mathematical understanding was conducted. Mathematical understanding was assessed with a range of scores from 0 to 4 based on Asfar, Asfar, Darmawati, & Darmawan (2018) and modified based on an analysis of knowledge in high-level thinking from Anderson, L.W., & Krathwohl (2001), i.e., conceptual understanding consists of classifying, comparing, and interpreting, while procedural understanding consists of executing and implementing. This capability is in accordance with the teaching material regarding inverse function in which competence is
obtained by students after learning about conceptual understanding which consists of the ability to identify, define definitions, associate, combine and merge inverse function formulas, while procedural understanding includes the ability to understand the rules of function, prove function identity, and solve problems according to the concept and principle of the inverse function.

Table 1. Scale point of analysis

<table>
<thead>
<tr>
<th>Aspect</th>
<th>Indicator</th>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conceptual Understanding</td>
<td>• Identify and determine the concept definitions/principles of inverse function</td>
<td>0</td>
<td>No attempt to understand the questions</td>
</tr>
<tr>
<td></td>
<td>• Associating, combining and merging concepts/principles of inverse function</td>
<td>1</td>
<td>Incorrect interpretation of all of the questions</td>
</tr>
<tr>
<td></td>
<td>• Carrying out calculations according to the concept/principle of inverse function</td>
<td>2</td>
<td>Incorrect interpretation of most of the questions</td>
</tr>
<tr>
<td></td>
<td>• Establishing and demonstrating the identity</td>
<td>3</td>
<td>Incorrect interpretation of a few of the questions</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>The correct interpretation of all of the questions</td>
</tr>
<tr>
<td>Procedural Understanding</td>
<td>• Establishing and demonstrating the identity</td>
<td>0</td>
<td>No attempt</td>
</tr>
<tr>
<td></td>
<td>• Establishing and demonstrating the identity</td>
<td>1</td>
<td>Inappropriate problem solving</td>
</tr>
<tr>
<td></td>
<td>• Establishing and demonstrating the identity</td>
<td>2</td>
<td>Few appropriate problem solving, mostly inappropriate</td>
</tr>
<tr>
<td></td>
<td>• Establishing and demonstrating the identity</td>
<td>3</td>
<td>Appropriate problem solving substantially, and less error and no reason</td>
</tr>
</tbody>
</table>

850
After going through the feasibility analysis of the test instrument, the test of the students' understanding on mathematics was performed at the pre-test and post-test in the control and experiment class to determine the differences of the students' understanding.

**Findings**

The elaboration of the Team Assisted Individualisation and Co-op Co-op learning model was implemented in an experimental class while Team Assisted Individualisation learning model was conducted in the control class. The implementation phase of the elaboration model is the giving and explanation of material, individual assignments, group discussions, presentation of tasks, evaluations, and awards, and conclusions of learning described as follows.
Figure 3. The implementation phase of elaboration on the learning model

Pre-test and Post-test Results

Figure 4 represents the average grade of pre-test and post-test of experimental and control class from this study. The results displayed that the experimental class had higher post-test grade compared with control class. This result describes that on the average the students' understanding of mathematics in experimental class is higher than the control class.
Figure 4. Grading comparison of pre-test and post-test on the understanding of mathematics

Hypothesis testing

Data obtained were analysed for the normality and homogeneity using SPSS® software assistance. The result showed that the samples were normally distributed and had homogenous variance. Moreover, the t-independent test was conducted to assess the parameter in the study.

Table 2. Hypothesis test of pre-test and post-test data

<table>
<thead>
<tr>
<th>Data</th>
<th>t</th>
<th>df</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-test</td>
<td>1.434</td>
<td>50</td>
<td>0.158</td>
</tr>
<tr>
<td>Post-test</td>
<td>6.678</td>
<td>50</td>
<td>0.001</td>
</tr>
</tbody>
</table>

The hypothesis test using t-independent test resulted that the grade of pretest from both experimental and control class had calculated t-value < t-table; 1.48 < 1.68; or α < p; 0.05 < 0.158, then H₀ is accepted.
Moreover, to find out the magnitude of the increase in the ability of students' understanding of mathematics, the analysis of normalised gain was conducted. The results of the analysis can be shown in the following Figure 5.

![The average N-gain value of the understanding of mathematics for the experimental and control class](image)

**Figure 5.** The average N-gain value of the understanding of mathematics for the experimental and control class

Based on the Figure above, the average values of N-gain on the students’ understanding of mathematics are 0.73 (high category) and 0.55 (medium category) for experimental and control class respectively.

**Indicator analysis of the students’ understanding of mathematics**

The results of the study were reinforced by the post-test answers carried out by the students. It is clearly seen the difference in the students' understanding between the experimental and the control class. The following figures, Figure 6 and 7, are the example of student answer for question number 3 and 6.
3. Determine the value $f^{-1}(4)$ from the function $f(x) = \frac{6x+2}{3x+5}$.

6. The function of $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ is determined by $f(x) = x+4$ and $g(x) = 3x-1$. Calculate the value of $(g \circ f)^{-1}(6)$.

Figure 6. The post-test answer of the experimental class

From the results of the experimental class answers, it appears that the student had solved the problem with the appropriate procedure in accordance with the concept/principle of inverse functions. It shows that the student had achieved the ability to understand the inverse function material conceptually. Whereas in the control class there are several students who answered similarly to as shown in Figure 7.
From Figure 7, it can be seen on the answer for number 3 that the students had answered the questions correctly, but the response was not complete according to the procedure of concept/principle for inverse functions. Moreover, the answer for number 6 indicated that the student had not correctly answered, and completed according to the method of concept/principle for inverse functions and also without any further explanation. Because the students had not been able to answer the questions based on conceptual and procedural understanding correctly and adequately, the students did not have good mathematical knowledge.

**Analysis of student preferences for the learning process**

Based on the results of the questionnaire in measuring student preferences seen from two aspects, namely the interest of students in learning mathematics and student interest in the learning process through the implementation of the TAI and Co-Op Co-Op elaboration models, showed that the average student preference for mathematics learning was 3.12 and the average preference of students through learning using model elaboration was of 3.34. Analysis of the
average student response preferences with 20 statements consisting of 5 indicators is shown in
the following table.

<table>
<thead>
<tr>
<th>No</th>
<th>Aspect</th>
<th>Indicator</th>
<th>The number of statements</th>
<th>Student Preference Score</th>
<th>Total average score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Students' attitudes towards mathematics</td>
<td>Showing interest in learning mathematics</td>
<td>2</td>
<td>3.04</td>
<td>3.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Demonstrate the application of studying mathematics</td>
<td>2</td>
<td>3.19</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Students' attitude towards the application of learning models</td>
<td>Demonstrate students' interest in learning mathematics by elaborating the TAI and Co-Op Co-Op model</td>
<td>11</td>
<td>3.19</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Showing interest in groups</td>
<td>2</td>
<td>3.25</td>
<td>3.34</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Demonstrate benefit in learning mathematics with the implementation of elaboration of the TAI and Co-Op Co-Op model</td>
<td>2</td>
<td>3.37</td>
<td></td>
</tr>
</tbody>
</table>

Average score 3.23
Discussion

Through the process of the implementation and the elaboration of the TAI and Co-Op Co-Op learning model, students appeared actively involved in constructing knowledge independently. The understanding category emphasised procedural understanding consisting of understanding the rules of function procedures, proving the identity of functions, and solving questions according to the concepts and principles of inverse function with the mathematical process explanation. Students who are low in understanding the concept and procedure in the problem will have an error selecting part of the information, using principles or formulas so it is less accurate in calculation to conclude well (Perbowo & Anjarwati, 2017).

In solving problems, students discussed, complemented, and gave input to the results of each group member by examining questions critically and creatively in finding the right problem solutions, so that the students' understanding was developed. This process emphasises the ability of students to associate, interpret, solve problems, conclude and unite various ideas/concepts, so a complete and comprehensive understanding is established.

Through examining the work of the students in the control and experiment class, it can be seen that there are differences between students who were taught using the elaboration of the TAI and Co-Op Co-Op learning models with students who were explained with the TAI learning model. Eventually, the grade of post-test from both the experimental and control class analysis using t-independent test showed that there is a difference in the ability of students' understanding of mathematics between the experimental and control class after the application of the learning model. The main reason is that the elaboration model of TAI and Co-Op Co-Op prioritises conceptual and procedural understanding in the process of constructing knowledge, through creative and critical thinking in solving problems, through collaboration and cooperation which ultimately impacts on the construction of students' independent knowledge. The process of communication exchanges of ideas/concepts in the problem-solving phase is conducted so that
students have a better understanding of the teaching materials. The existence of activities of exploring and thinking in learning will help students to achieve higher scores in mathematics, so that it can help the students to build trust in themselves in understanding and learning the new concept of mathematics (Khoule, Bonsu, & Houari, 2017).

The average preferences of the students after learning using the elaboration of TAI and Co-Op Co-Op learning model is a positive response (3.23). The students' preference indicated that the elaboration of the TAI and Co-Op Co-Op learning model could provide a better learning process to develop students' understanding of mathematics compared to another learning model. Thus the elaboration of the TAI and Co-Op Co-Op learning model has become an alternative solution in fostering the students' understanding of mathematics which accommodates 21st century skills in the Education era 4.0 namely 4C (critical thinking, creative thinking, collaboration, and communication). Twenty-first century skills are very important to facilitate students in life in global communities, life that rapidly changes and also as a key element in teaching and learning that will meet the demands of a knowledge-based workforce (Warner & Kaur, 2017).

Conclusion

Based on the research, the results of the t-independent test obtained for the elaboration of learning model was calculated where t-value > t-table; 6.68 > 1.68; or α > p; 0.05 > 0.001. It provided that there is a difference in the ability of students' mathematical understanding between the experimental class and the control class after the application of the learning model. The increase in the students' understanding of mathematics between experimental class students and control class represented by the value of N-gain is 0.73 (high category) and 0.55 (medium category), respectively. These results were confirmed by students' preference after the learning process with a positive response or 3.23. The implementation of the elaboration on the TAI and
Co-Op Co-Op learning models can improve the students' understanding of mathematics by focusing on the knowledge construction process through creative and critical thinking processes and communication processes, through exchanging ideas/concepts in collaborative problem-solving processes so that students have a better understanding of the teaching materials.

**Suggestions**

Mathematics learning is still very much dominated by learning, which only emphasises being able to solve problems without understanding the procedures and concepts of the problem being worked on. As a result, the teaching material is not able to last long in the memory of the students, so the giving of questions that are different from the examples that have been given, the students will have difficulty, being about students' ability to overcome mathematical difficulties. Problem solving ability is the most important point because it is related with ability to understand and solve solutions towards problems in everyday life (Asfar, Nur, & Asfar, 2019). Besides that, mastery of several learning models should be possessed by the teacher in learning the material in the classroom, but time and curiosity become the limitations of the teacher in applying and looking for the right learning model in improving students' understanding in learning mathematics.

The incorporation of the Co-op Co-op learning model with the TAI learning model becomes an alternative solution to answer problems in how learning materials are mastered and understood by students. The incorporation of this learning model seeks to correct the weaknesses of the two models so that in its implementation it emphasises conceptual understanding and procedural understanding. Both of these understandings, if facilitated and emphasised in learning, will enhance students' mathematical achievements. Besides that, the emphasis of both conceptual and procedural understanding, integrated with 21st century skills, is expected to produce graduates that are needed later in the future.
References


